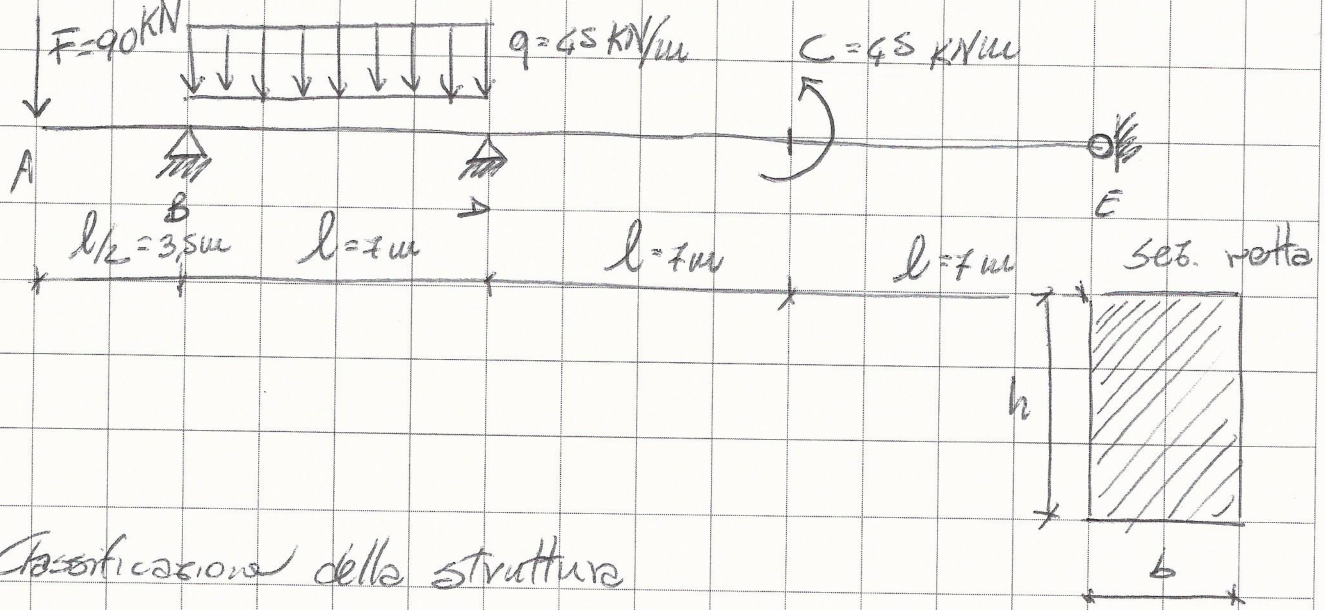
Esercizio - Trave continua e verifica di sicurezza

(1)



1) Classificazione della struttura

$$l - i = 3t - s$$ (problema piano - 3 g.d.l. (estensionale e flessionale))

$$3t = 3$$

$s = 6$ (sono 3 cerniere con molteplicità 2)

$$l = 0 \Rightarrow 0 - i = 3 - 6 \Rightarrow i = 3$$

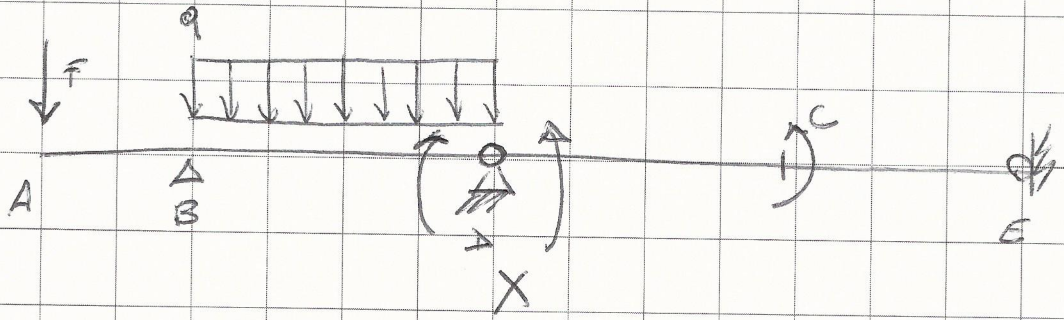
Considerando che non vi sono forze esterne di tipo estensionale si possono considerare solo 2 g.d.l. flessionali (abbassamenti e rotazioni) e il vincolo delle cerniere solo in direzione ortogonale alla trave (il risultato del problema estensionale è nullo)

$$l - i = 2t - s \Rightarrow 0 - i = 2 - 3 \Rightarrow i = 1$$

2

(2) Valutazione delle reazioni vincolari

Struttura 1 volta iperstatica → creare 1 sconnessione e inserire l'incognita iperstatica duale



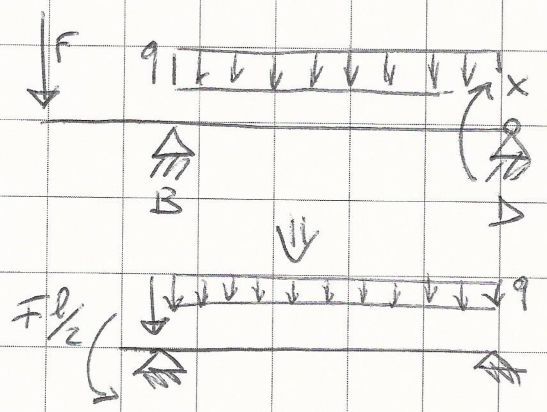
Si va a ristabilire la congruenza delle rotazioni in Δ (a sinistra e destra) per valutare l'incognita iperstatica

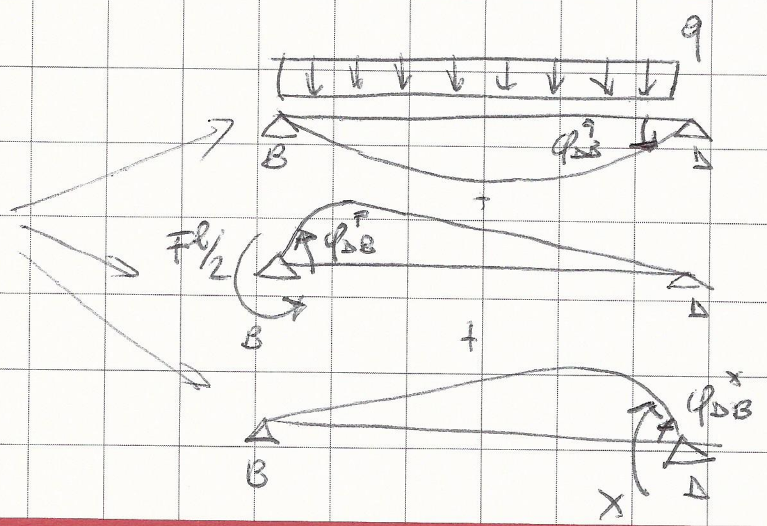
$$\varphi_{\Delta B} = \varphi_{\Delta E}$$

sovrapposizione degli effetti

$$\varphi_{\Delta B} = \varphi_{\Delta B}^{(x)} + \varphi_{\Delta B}^{(q)} + \varphi_{\Delta B}^{(F)}$$

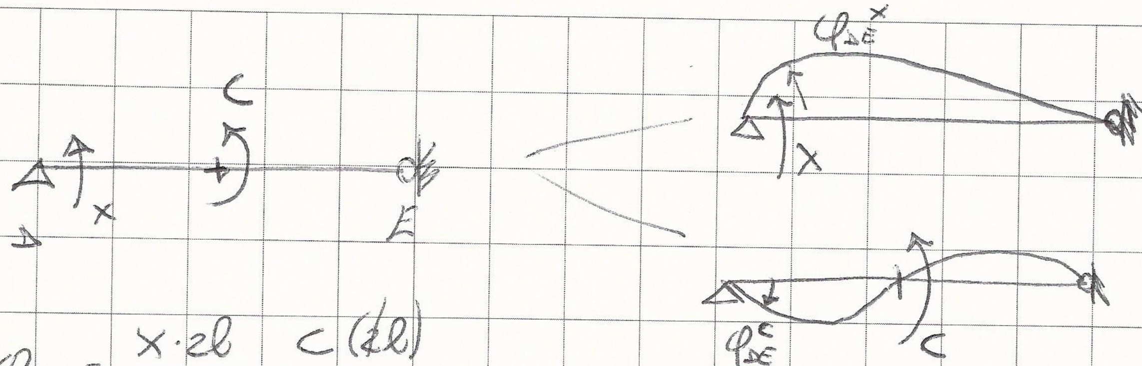
$$\boxed{\varphi_{\Delta E} = \varphi_{\Delta E}^{(x)} + \varphi_{\Delta E}^{(C)}}$$ pag. 3





$$\varphi_{\Delta B} = -\frac{X l}{3EI} + \frac{q l^3}{24EI} - \frac{Fl}{2} \cdot \frac{l}{6EI}$$

rot. ↻ positive



$$\varphi_{DE} = \frac{X \cdot 2l}{3EI} - \frac{C(2l)}{\cancel{24}EI}$$
12

$$\varphi_{DB} = \varphi_{DE}$$

$$\Downarrow$$

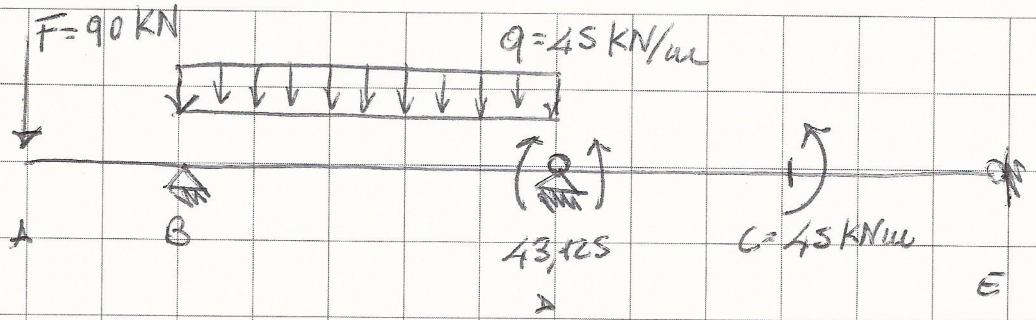
$$-\frac{X\cancel{l}}{\cancel{3}EI} + \frac{q l^{\cancel{3}\,2}}{\cancel{24}EI}_{8} - \frac{F l^{\cancel{2}}}{\cancel{12}EI}_{4} = \frac{2X\cancel{l}}{\cancel{3}EI} - \frac{C\cancel{l}}{\cancel{12}EI}_{4}$$

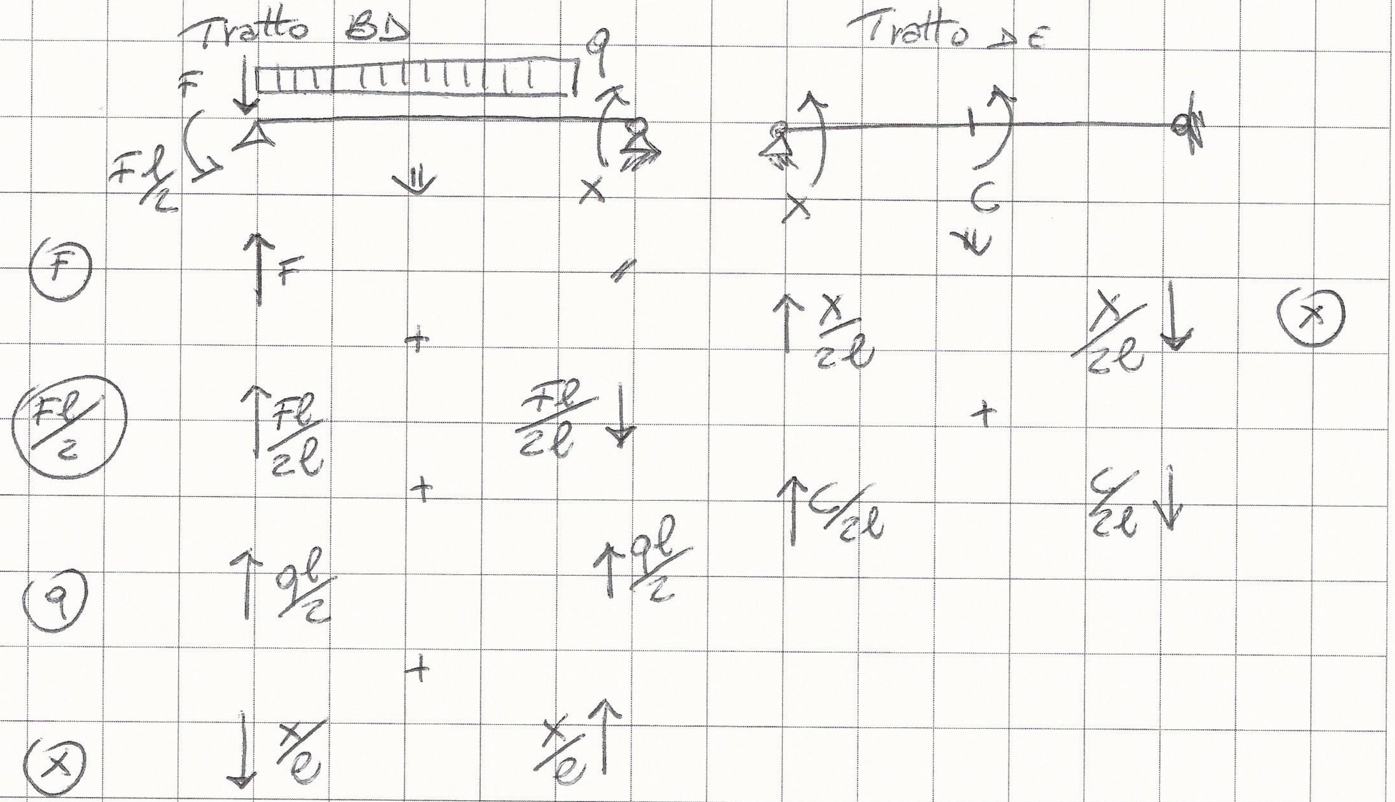
$$-X + \frac{ql^2}{8} - \frac{Fl}{4} = 2X - \frac{C}{4}$$

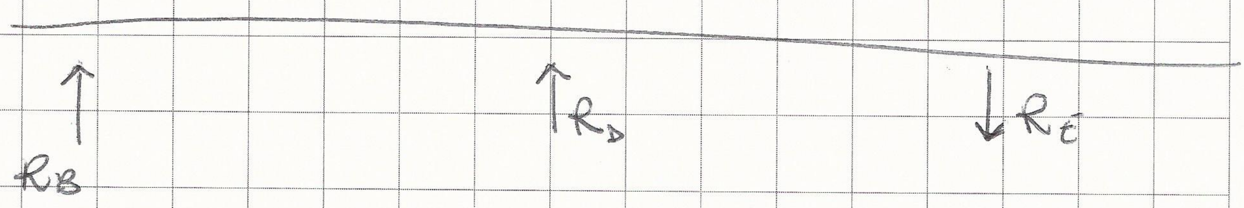
$$3X = \frac{ql^2}{8} - \frac{Fl}{4} + \frac{C}{4}$$

$$X = \frac{1}{3}\left[\frac{45 \cdot 7^2}{8} - \frac{90 \cdot 7}{4} + \frac{45}{4}\right] = \frac{2205 - 1206 + 90}{24} = \frac{345}{8} = 43{,}125\ kN/m$$

Procediamo al calcolo delle reazioni vincolari sempre con la sovrapposizione degli effetti.



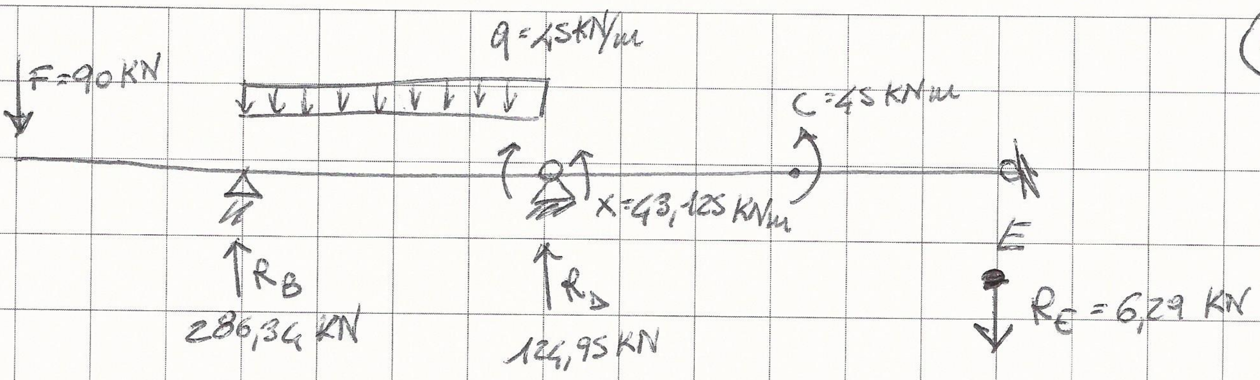




$$R_B = F + \frac{F}{2} + \frac{ql}{2} - \frac{X}{l} = 90+45+157{,}5-6{,}16 = 286{,}34 \text{ kN}$$

$$R_D = -\frac{F}{2} + \frac{ql}{2} + \frac{X}{l} + \frac{X}{2l} + \frac{C}{2l} = 124{,}95 \text{ kN}$$

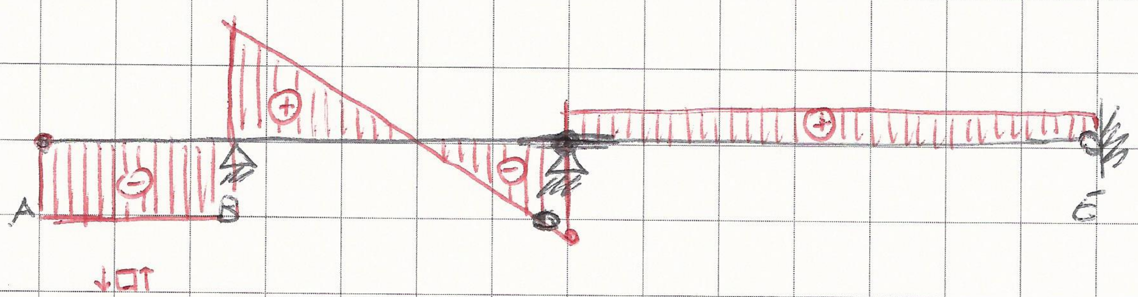
$$R_E = \frac{X}{2l} + \frac{C}{2l} = 6{,}29 \text{ kN}$$



③ Diagrammi delle sollecitazioni ~~N~~, T, M

0

T



$T_A = -F = -90$ KN

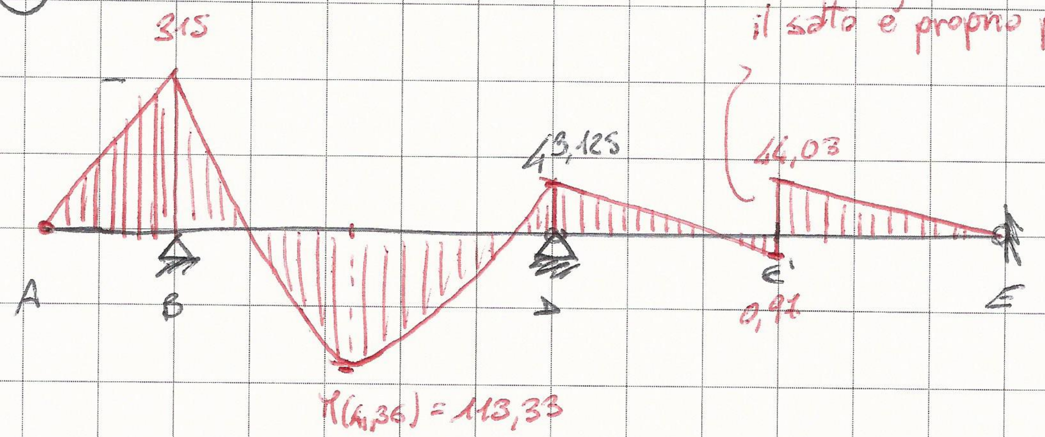
$T_B^s = T_A = -90$ KN $\qquad T_B^d = T_B^s + R_B = 196,34$ KN

$T_D^s = T_B^d - q \cdot l = -118,66$ KN $\qquad T_D^d = T_D^s + R_D = 6,29$ KN

$T_E = R_E = 6,29$ KN

M

il salto é proprio pari a C



$M_A = 0$

$M_E = 0$

$M_B^s = F \cdot l/2 = 315 \text{ KNm}$

$M_B^D = M_B^s = 315 \text{ KNm}$

$M_D^s = F(l/2 + l) - R_B \cdot l + \frac{q l^2}{2} = 43{,}125 \text{ KNm} = X$

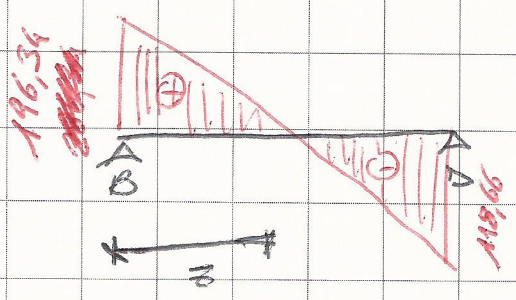
procediamo da destra

$M_{C'}^D = R_E \cdot l = 44{,}03$

$M_{C'}^s = R_E \cdot l - C = -0{,}97 \text{ KN}$

Calcolo z di Mmax in campata BD

Guardando il taglio



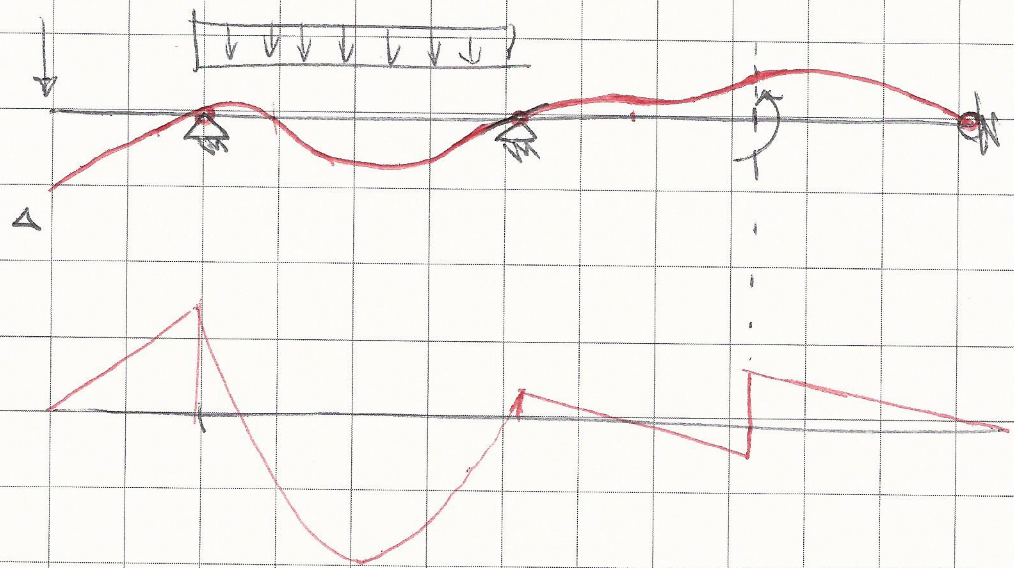
$$\frac{196{,}34}{z} = \frac{196{,}39 + 118{,}66}{7\,m}$$

$\Downarrow$

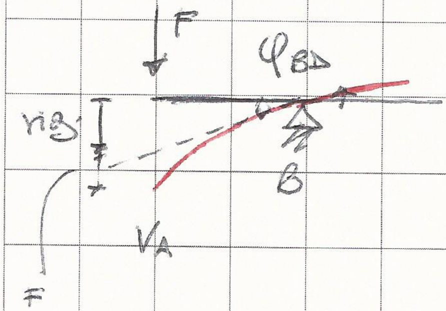
$z = 4{,}36\,m$

$$M(z) = F \cdot \left(\frac{l}{2} + 4{,}36\right) - R_B \cdot 4{,}36 + \frac{q \cdot 4{,}36^2}{2} = -113{,}33$$

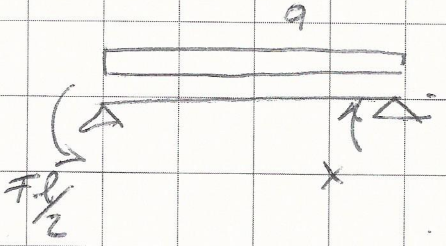
(4) Disegno della deformata a maniere



(5) Valutazione dello spostamento in A



somma di una rotazione rigida dell'estremo B e dell'abbassamento dell'estremità della mensola dovuta a F

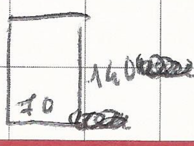


$$\varphi_{BA} = \varphi_{BA}^{(F)} + \varphi_{BA}^{(q)} + \varphi_{BA}^{(x)}$$

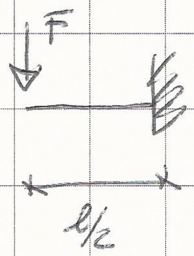
$$\varphi_{BA} = \frac{Fl}{2}\,\frac{l}{3EI} - \frac{ql^3}{24EI} + \frac{Xl}{6EI} = 0{,}0423 \text{ rad} = 2{,}4236°$$

$E = 210\,000 \ N/mm^2$

$I = \frac{bh^3}{12} = \frac{70 \cdot 140^3}{12} = 16\,006\,666{,}67 \ mm^4$



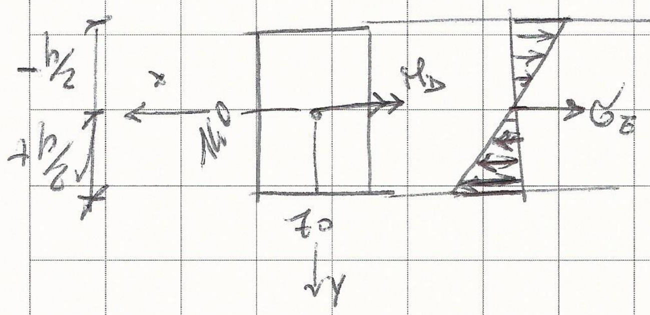
$$V_A^{(F)} = \frac{F l^3}{3EI} \Rightarrow \frac{F \cdot (l/2)^3}{3EI}$$



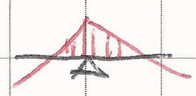
$$V_A^{(F)} = 382,65 \text{ mm}$$

$$V_A^{(Tot)} = V_A^{(F)} + \varphi_{B\Delta} \cdot l/2 = 382,65 + 0,0423 \cdot 3500 = 530,7 \text{ mm}$$

⑥ Tensione normale massima e minima nella sezione Δ



$M_\Delta = X = -43,125$ kNm



$$\sigma_z = \frac{N}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$N_z = 0$

→ termine flessione fuori dal piano del disegno

$$\sigma_z = \frac{M_x}{I_x} y$$

$$\sigma_z(z = -h/2) = -\frac{43,125}{1,6 \cdot 10^5}\left(\frac{-140}{2}\right) = 188,67 \text{ N/mm}^2$$

$$\sigma_z(z = h/2) = -\frac{43,125}{1,6 \cdot 10^5}\left(\frac{140}{2}\right) = 188,67 \text{ N/mm}^2$$

Verifica di resistenza

$$\sigma_{eq} = \sigma_z = 188,67 > \sigma_{adm} = \frac{235}{1,5} = 160 \text{ N/mm}^2$$

(Fe360)

← coeff. sic.

Non verificata