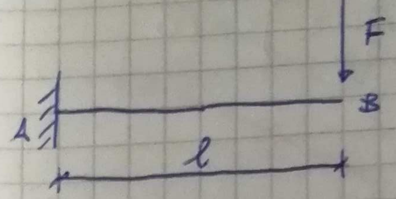
25/05/2004

Esercizio



$$\frac{d^4 v}{dz^4} = 0 \qquad v(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3$$

$$z=0 \begin{cases} v(0)=0 & \Rightarrow \boxed{a_0 = 0} \\ \varphi(0)=0 & \Rightarrow \boxed{a_1 = 0} \end{cases}$$

$$z=l \begin{cases} T(l)=F & -EI\,6a_3 = F \\ M(l)=0 & M(z) = -EI(2a_2 + 6a_3 z) \Rightarrow -EI(2a_2 + 6a_3 l) = 0 \end{cases}$$

$$\boxed{a_3 = -\frac{F}{6EI}} \qquad \boxed{a_2 = \frac{Fl}{2EI}}$$

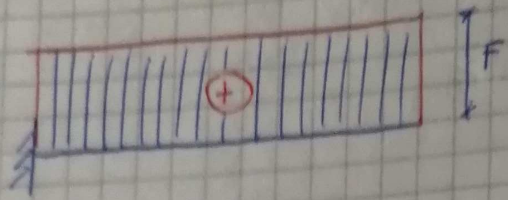
$$v(z) = \frac{Fl}{2EI}\left[z^2 - \frac{z^3}{3l}\right] = \frac{Fl^3}{2EI}\left[\left(\frac{z}{l}\right)^2 - \frac{1}{3}\left(\frac{z}{l}\right)^3\right]$$

$$\varphi(z) = -\frac{Fl^2}{2EI}\left[2\frac{z}{l} - \left(\frac{z}{l}\right)^2\right]$$

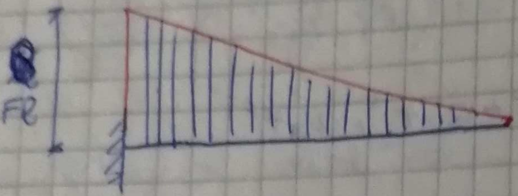
$$M(z) = -\frac{Fl}{2}\left[2 - 2\frac{z}{l}\right] = -Fl\left[1 - \frac{z}{l}\right]$$

$$T(z) = F$$

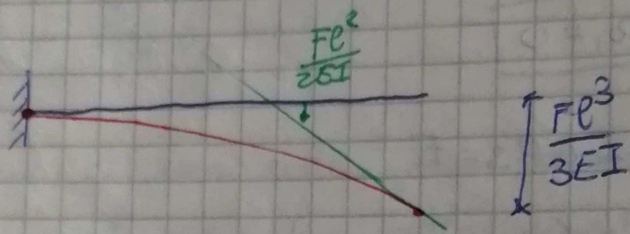
(T)



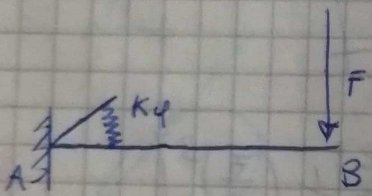
(M)



(V)



Se l'incastro è cedevole elasticamente rispetto alla rotazione



$$\varphi = -\frac{M}{K\varphi}$$

$$z=0 \begin{cases} v(0)=0 \\ \varphi(0) = -\dfrac{M(0)}{K\varphi} \end{cases} \qquad a_0 = 0$$

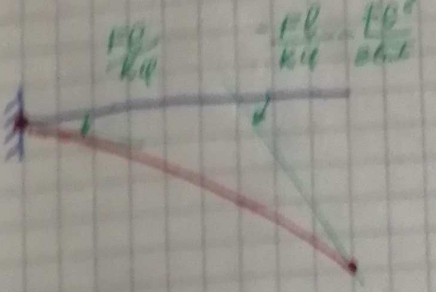
$$z=l \begin{cases} T(l)=F \\ M(l)=0 \end{cases} \qquad \left.\begin{matrix} a_2 = \\ a_3 = \end{matrix}\right\} \text{stessi di prima}$$

$$-a_1 = -\frac{1}{K\varphi}\left(-EI\, 2a_2\right) \Rightarrow a_1 = -\frac{1}{K\varphi} Fl$$

- $v(z) = +\frac{Fl}{K\varphi} z + \frac{Fl}{2EI} z^2 - \frac{F}{6EI} z^3$

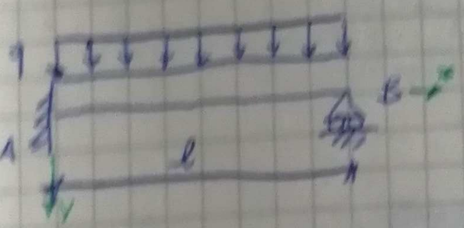
(I) ed (II) resteranno invariati

(4)



$$\frac{Fl^3}{3EI} + \frac{Fl^2}{K_4}$$

Esercizio



$$\frac{d^4v}{dz^4} = \frac{q}{EI} \qquad v(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \frac{q}{24EI} z^4$$

$$z=0 \begin{cases} v(0) = 0 & \rightarrow \boxed{a_0 = 0} \\ \varphi(0) = 0 & \rightarrow \boxed{a_1 = 0} \end{cases}$$

$$z=l \begin{cases} v(l) = 0 & a_2 l^2 + a_3 l^3 + \frac{ql^4}{24EI} = 0 \\ M(l) = 0 \end{cases}$$

$$M(l) = -EI\left(2a_2 + 6a_3 l + \frac{ql^2}{EI}\right) = 0$$

Risolvendo il sistema

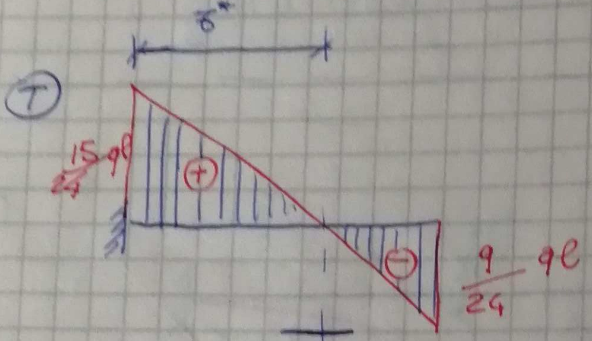
$$\boxed{a_2 = \frac{3}{48}\frac{ql^2}{EI}} \qquad \boxed{a_3 = -\frac{5}{48}\frac{ql}{EI}}$$

$$v(z) = \frac{ql^4}{48EI}\left[3\left(\frac{z}{l}\right)^2 - 5\left(\frac{z}{l}\right)^3 + \left(\frac{z}{l}\right)^4\right]$$

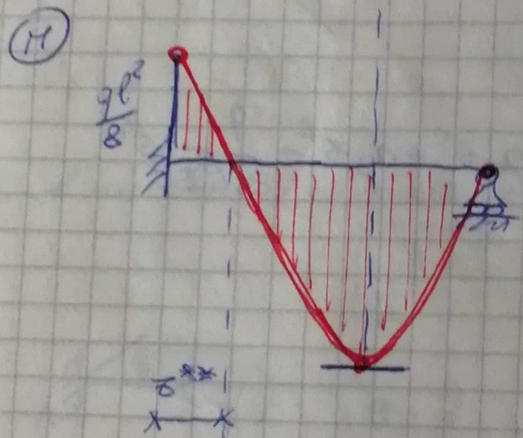
- $\varphi(z) = -\frac{ql^3}{48EI}\left[6\frac{z}{l} - 15\left(\frac{z}{l}\right)^2 + 8\left(\frac{z}{l}\right)^3\right]$

- $M(z) = -\frac{ql^2}{24}\left[3 - 15\frac{z}{l} + 12\left(\frac{z}{l}\right)^2\right]$

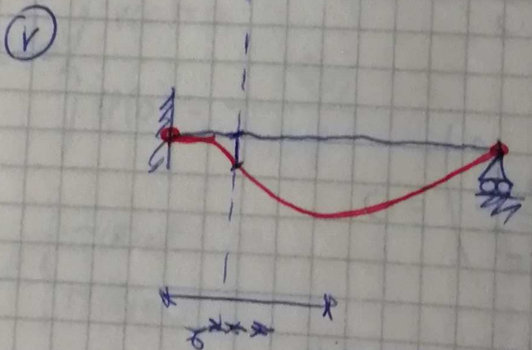
$T(z) = -\frac{ql}{24}\left[-15 + 24\frac{z}{l}\right]$



$z^* = \frac{15}{24}l$



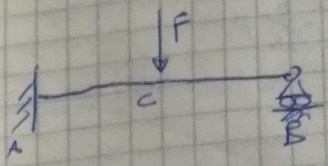
$M(z^*) \cong 0,79\, l^2$



$z^{**} = 0,25\, l$

imporre $\varphi(z) = 0$

Metodo delle forze o della congruenza



TRATTO AC

$$v^{(1)}(z) = \cancel{a_0 + a_1 z} + a_2 z^2 + a_3 z^3$$

TRATTO CB

$$v^{(2)}(z) = b_0 + b_1 z + b_2 z^2 + b_3 z^3$$

$$z=0 \begin{cases} v^{(1)}(0)=0 \Rightarrow \boxed{a_0=0} \\ \varphi^{(1)}(0)=0 \Rightarrow \boxed{a_1=0} \end{cases}$$

$$z=\ell \begin{cases} v^{(2)}(\ell)=0 \Rightarrow b_0+b_1\ell+b_2\ell^2+b_3\ell^3=0 \\ M^{(2)}(\ell)=0 \Rightarrow -EI(2b_2+6b_3\ell)=0 \end{cases}$$

$$z=\frac{\ell}{2} \begin{cases} v^{(2)}(\ell/2)=v^{(1)}(\ell/2) \quad \Delta v=0 \Rightarrow v^{(2)}(\ell/2)-v^{(1)}(\ell/2)=0 \\ \Delta\varphi=0 \Rightarrow \varphi^{(2)}(\ell/2)-\varphi^{(1)}(\ell/2)=0 \\ \Delta M=0 \Rightarrow M^{(2)}(\ell/2)-M^{(1)}(\ell/2)=0 \\ \Delta T=-F \Rightarrow T^{(2)}(\ell/2)-T^{(1)}(\ell/2)=-F \end{cases}$$

$$a_2=\frac{9}{96}\frac{F\ell}{EI} \qquad a_3=-\frac{11}{96}F$$

$$b_0=-\frac{F\ell^3}{48EI} \qquad b_1=\frac{F\ell^2}{8EI} \qquad b_2=-\frac{5}{32}\frac{F\ell}{EI} \qquad b_3=\frac{5}{96}F$$

$$v^{(1)}(z)=\frac{F\ell^3}{96EI}\left[9\left(\frac{z}{\ell}\right)^2-11\left(\frac{z}{\ell}\right)^3\right]$$

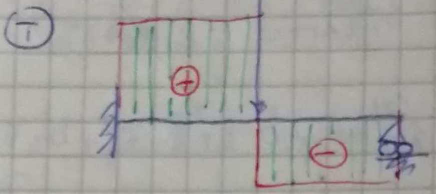
$$\varphi^{(1)}(z)=-\frac{F\ell^2}{\cancel{96}\,32\,EI}\left[\cancel{18}\,6\,\frac{z}{\ell}-\cancel{33}\,11\left(\frac{z}{\ell}\right)^2\right]$$

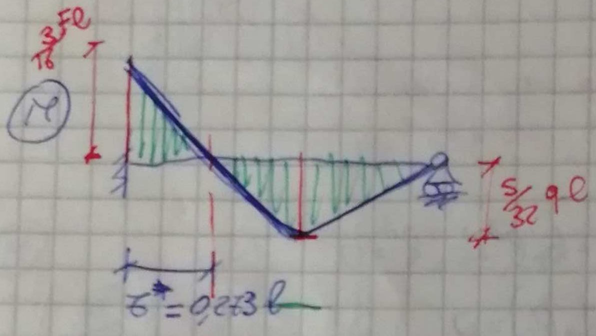
$$v^{(2)}(z)=\frac{F\ell^3}{8EI}\left[-\frac{1}{6}+\frac{z}{\ell}-\frac{5}{4}\left(\frac{z}{\ell}\right)^2+\frac{5}{12}\left(\frac{z}{\ell}\right)^3\right]$$

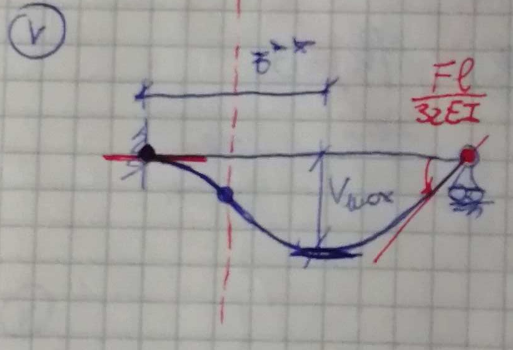
$$\varphi^{(2)}(z)=-\frac{F\ell^2}{8EI}\left[1-\frac{5}{2}\frac{z}{\ell}+\frac{5}{4}\left(\frac{z}{\ell}\right)^2\right]$$

$M^{(1)}(z) = -\frac{Fl}{16}\left[3 - 11\frac{z}{l}\right]$ $\qquad$ $M^{(2)}(z) = -\frac{Fl}{8}\left[-\frac{5}{2} + \frac{5}{2}\frac{z}{l}\right]$

$T^{(1)}(z) = \frac{F}{16}11$ $\qquad$ $T^{(2)}(z) = -\frac{5}{16}F$



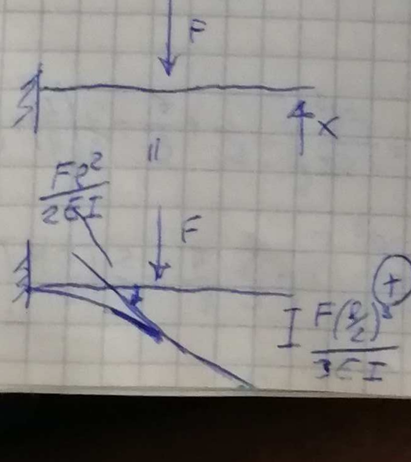




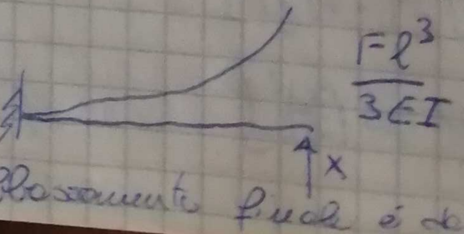
$\varphi^{(2)}(z) = 0 \quad \Rightarrow z^{**} = 0,54\,l$

$V_{max} = v^{(2)}(z^{**})$

Determinare reazioni vincolari con il metodo delle forze



$v_B^{(F)} + v_B^{(x)} = 0$



e l'abbassamento finale è dato da

$$\frac{F\left(\frac{l}{2}\right)^3}{3EI} + \frac{Fl^2}{2EI}\cdot\frac{l}{2}$$

Nelle travi iperstatiche i cedimenti vincolari provocano sollecitazioni interne.

$$V(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3$$

$$z=0 \begin{cases} v(0)=0 & \Rightarrow a_0 = 0 \\ \varphi(0) = 0 & \Rightarrow a_1 = 0 \end{cases}$$

$$z=l \begin{cases} v(l) = V_B^* & a_2 l^2 + a_3 l^3 = V_B^* \\ M(l) = 0 & -EI(2a_2 + 6a_3 l) = 0 \end{cases} \quad \Rightarrow a_3 = -\frac{V_B^*}{2l^3} \quad a_2 = -3a_3 l \quad a_2 = \frac{3}{2}\frac{V_B^*}{l^2}$$

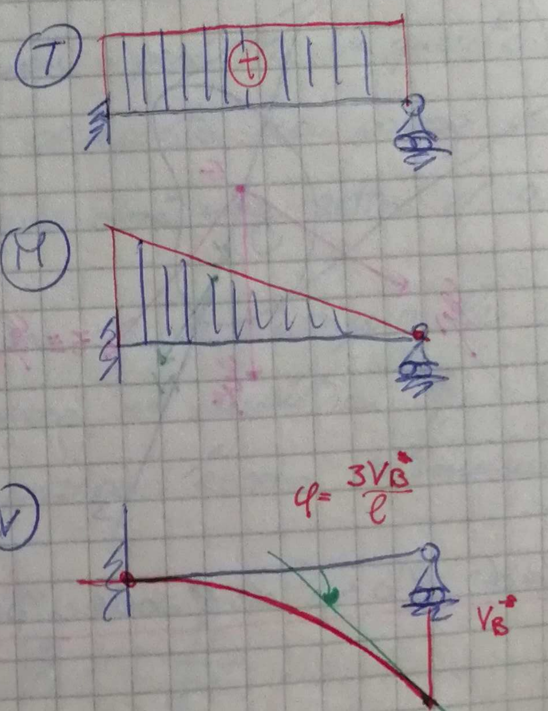
$$V(z) = \frac{3}{2}\frac{V_B^*}{l^2} z^2 - \frac{V_B^*}{2l^3} z^3$$

$$V(z) = \frac{V_B^*}{2}\left[3\left(\frac{z}{l}\right)^2 - \left(\frac{z}{l}\right)^3\right]$$

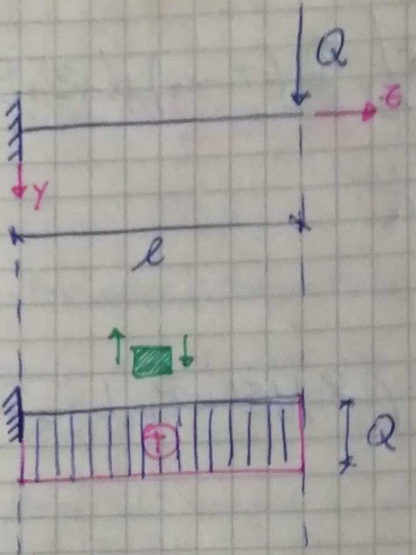
$$\varphi(z) = -\frac{3V_B^*}{2l}\left[2\frac{z}{l} - \left(\frac{z}{l}\right)^2\right]$$

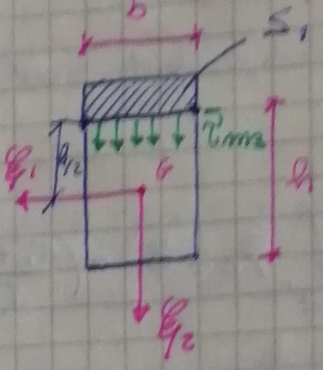
$$M(z) = -EI\,\frac{3V_B^*}{l^2}\left[1 - \frac{z}{l}\right]$$

$$T(z) = +EI\,\frac{3V_B^*}{l^3}$$



Sezione rettangolare





$$\bar\tau_{m3} = -\frac{T_2\, S_1'}{b\, I_1}$$

$$I_1 = \frac{bh^3}{12}$$

$$S' = b\left(\frac{h}{2}+\xi_2\right)\cdot\left[\xi_2 - \frac{1}{2}\left(\frac{h}{2}+\xi_2\right)\right] = b\left(\frac{h}{2}+\xi_2\right)\left[\xi_2 - \frac{h}{4} - \frac{\xi_2}{2}\right]$$

$$S' = b\left(\frac{h}{2}+\xi_2\right)\left(\frac{\xi_2}{2}-\frac{h}{4}\right) = \frac{b}{2}\left(\xi_2+\frac{h}{2}\right)\left(\xi_2-\frac{h}{2}\right)$$

$$S' = \frac{b}{2}\left(\xi_2^2 - \frac{h^2}{4}\right)$$

$$\tau_{m3} = -\frac{T}{\frac{b^{\cancel{2}}h^3}{\cancel{12}\,6}}\,\frac{\cancel{b}}{\cancel{2}_1}\left(\xi_2^2-\frac{h^2}{4}\right)$$

$$\boxed{\tau_{m3} = -\frac{6T}{bh^3}\left(\xi_2^2-\frac{h^2}{4}\right)}$$

$$\bar\tau_{m3}(0) = \frac{3}{2}\frac{T}{bh}$$