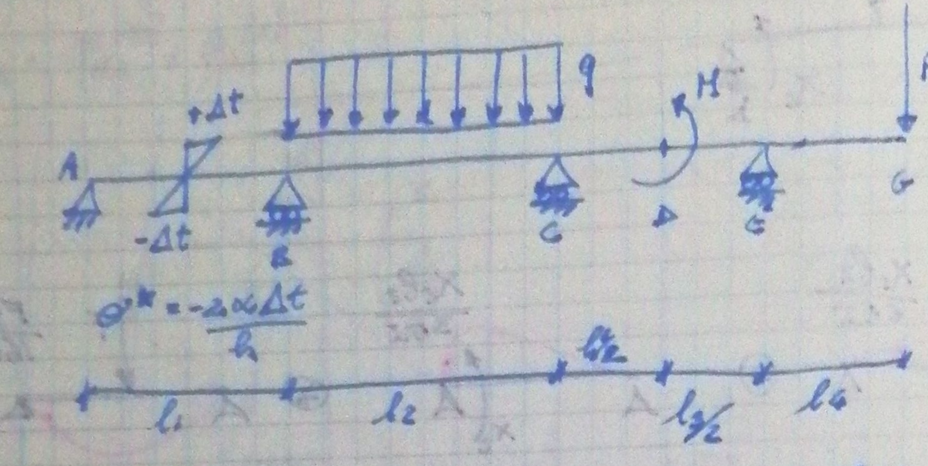
Compito 1

Profilato IPE 300 Fe 360



$2{,}1 \cdot 10^8$ KN/m²

$l_1 = 4$ m
$l_2 = 6$ m
$l_3 = 4$ m
$l_4 = 2$ m

$q = 20$ KN/m
$M = 10$ KNm
$F = 28$ KN
$\Delta t = 15°C$

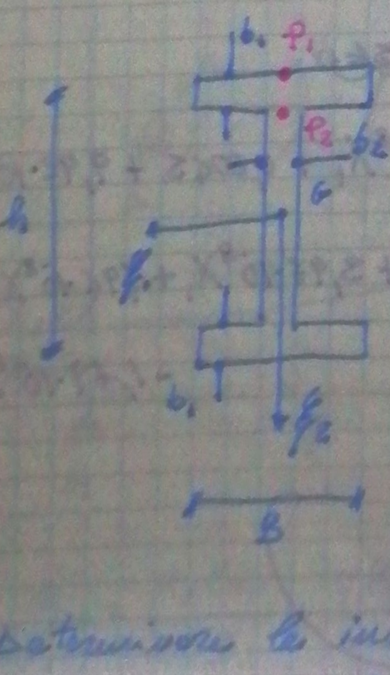
$E = 2{,}1 \cdot 10^{-5}$ N/mm² $= 2{,}1 \cdot 10^{11}$ N/m²

$\alpha = 1{,}2 \cdot 10^{-5}$ °C⁻¹

$\sigma_{adm} = 160$ N/mm² $= 16000$ N/cm²

$A = 51{,}88$ cm² $I = I_x = 7999$ cm⁴ $= 7999 \cdot 10^{-8}$ m⁴

$W_x = 533{,}27$ cm³



h = 300 mm
B = 150 mm
$b_1 = 10{,}7$ mm
$b_2 = 7{,}1$ mm

$P_1 = (0, 150)$
$P_2 = (0, -139{,}3)$

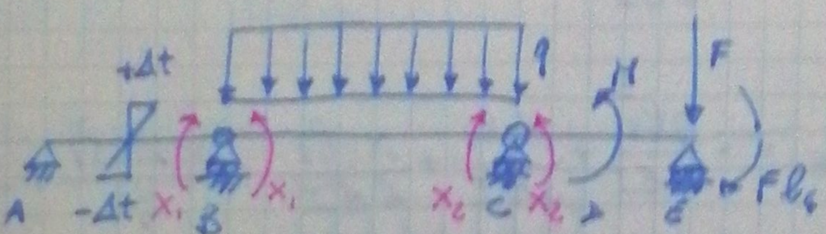
- Determinare le incognite iperstatiche
- Le reazioni vincolari
- Diagrammi T ed M
- Andamento linea elastica

Spostamento v nella mezzeria di BC e alla punta dello sbalzo G (si considera come una mensola incastrata con cedimento φ_E.

- Verifica di resistenza con criteri Tresca e Von Mises, nei punti P_1 e P_2 della sezione più sollecitata a flessione e taglio.

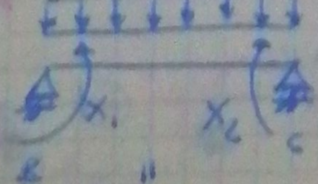
$3t = 3$

$s = 4$ $\quad i = 2$

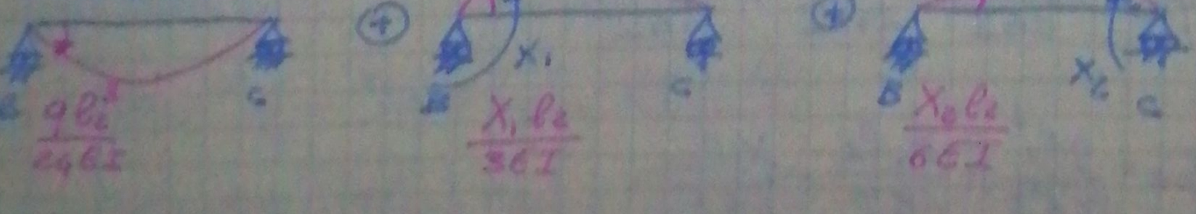


$$\begin{cases} \Delta\varphi_B = \varphi_{BC} - \varphi_{BA} = 0 \\ \Delta\varphi_C = \varphi_{CE} - \varphi_{CB} = 0 \end{cases}$$

φ_{BC}?



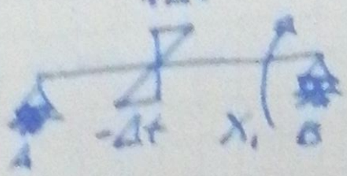
$$\varphi_{BC} = \varphi_{BC}^{(q)} + \varphi_{BC}^{(X_1)} + \varphi_{BC}^{(X_2)}$$



$$\frac{q l_2^3}{24EI} \qquad \frac{X_1 l_2}{3EI} \qquad \frac{X_2 l_2}{6EI}$$

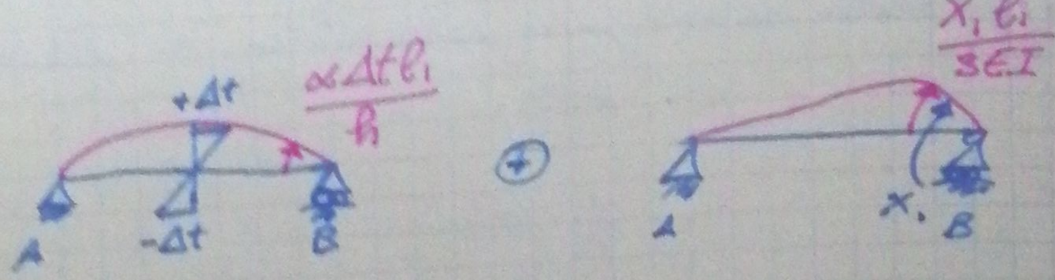
$$\varphi_{BC} = -\frac{q l_2^3}{24EI} + \frac{X_1 l_2}{3EI} + \frac{X_2 l_2}{6EI}$$

$\varphi_{BA} = ?$



$$\varphi_{BA} = \varphi_{BA}^{(\Delta t)} + \varphi_{BA}^{(X_1)}$$

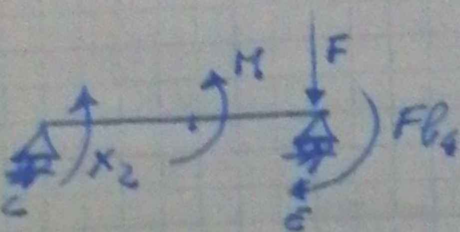
=



$$\varphi_{BA} = -\frac{\alpha \Delta t\, l_1}{h_1} - \frac{X_1 l_1}{3EI}$$

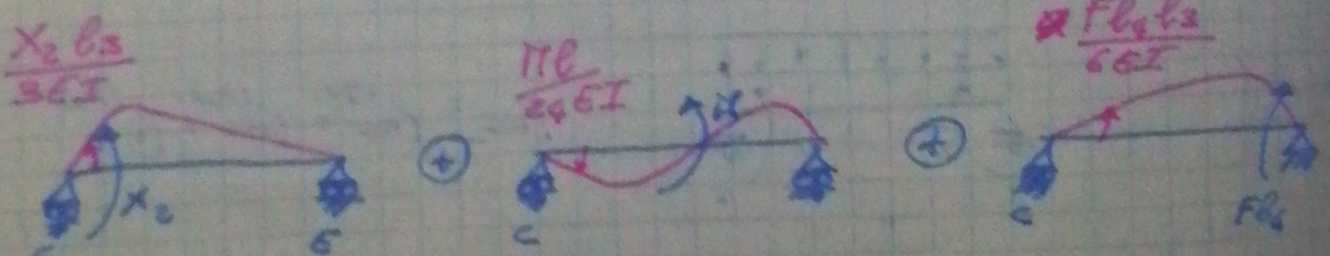
$$\Delta\varphi_B = \varphi_{BC} - \varphi_{BA} = -\frac{q l_2^3}{24EI} + \frac{X_1 l_2}{3EI} + \frac{X_2 l_2}{6EI} + \frac{\alpha \Delta t\, l_1}{h_1} + \frac{X_1 l_1}{3EI} = 0$$

$\varphi_{CE} = ?$



$$\varphi_{CE} = \varphi_{CE}^{(X_2)} + \varphi_{CE}^{(M)} + \varphi_{CE}^{(Fl_4)}$$

=



$$\varphi_{CE} = \frac{X_2 l_3}{3EI} - \frac{M l_3}{24EI} + \frac{F l_4 l_3}{6EI}$$

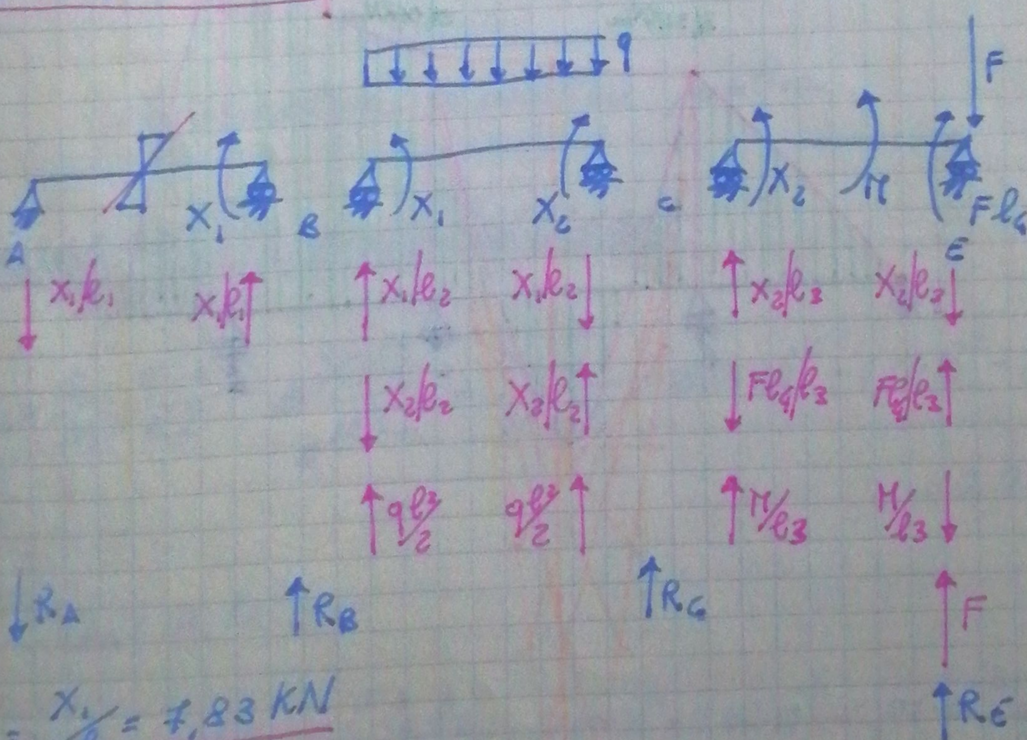
$$\varphi_{CB} = \frac{q l_2^3}{24EI} - \frac{X_1 l_2}{6EI} - \frac{X_2 l_2}{3EI}$$

$$\Delta\varphi_C = \varphi_{CE} - \varphi_{CB} = \frac{X_2 l_3}{3EI} - \frac{M l_3}{24EI} + \frac{F l_4 l_3}{6EI} - \frac{q l_2^3}{24EI} + \frac{X_1 l_2}{6EI} + \frac{X_2 l_2}{3EI} = 0$$

$$\begin{cases} -1{,}07\cdot10^{-2} + 1{,}19\cdot10^{-4}X_1 + 5{,}95\cdot10^{-5}X_2 + 0{,}0024 + 7{,}94\cdot10^{-5}X_1 = 0 \\ 7{,}94\cdot10^{-6}X_2 - 9{,}92\cdot10^{-5} + 1{,}98\cdot10^{-3} - 1{,}07\cdot10^{-2} + 5{,}95\cdot10^{-5}X_1 + 1{,}19\cdot10^{-4}X_2 = 0 \end{cases}$$

$$\begin{cases} 19{,}84\cdot10^{-5}X_1 + 5{,}95\cdot10^{-5}X_2 = 8{,}3\cdot10^{-3} \\ 5{,}95\cdot10^{-5}X_1 + 19{,}84\cdot10^{-5}X_2 = 8{,}82\cdot10^{-3} \end{cases}$$

$$\begin{cases} \underline{X_1 = 31{,}32\ kNm} \\ \underline{X_2 = 35{,}06\ kNm} \end{cases}$$



$R_A = X_1/l_1 = \underline{7{,}83\ kN}$

$R_B = X_1/l_1 + X_1/l_2 - X_2/l_2 + \dfrac{ql_2}{2} = \underline{67{,}21\ kN}$

$R_C = -X_1/l_2 + X_2/l_2 + \dfrac{ql_2}{2} + X_2/l_3 - \dfrac{Fl_4}{l_3} + M/l_3 = \underline{59{,}39\ kN}$

$R_E = -X_2/l_3 + \dfrac{Fl_4}{l_3} - M/l_3 + F = \underline{26{,}24\ kN}$

Diagrama T

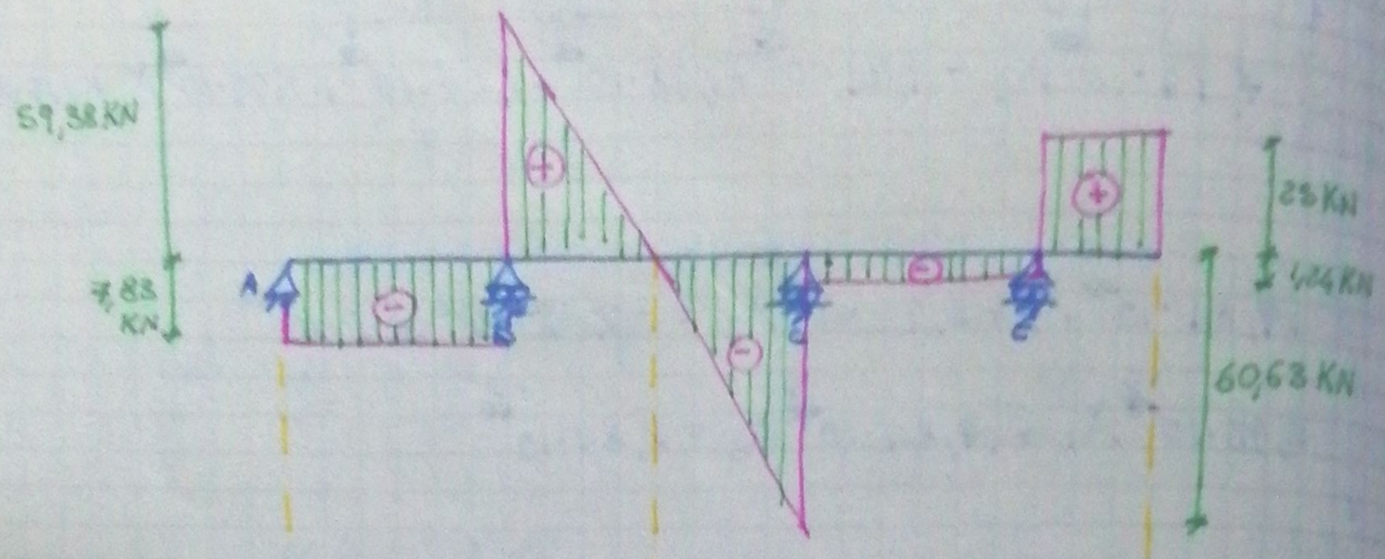
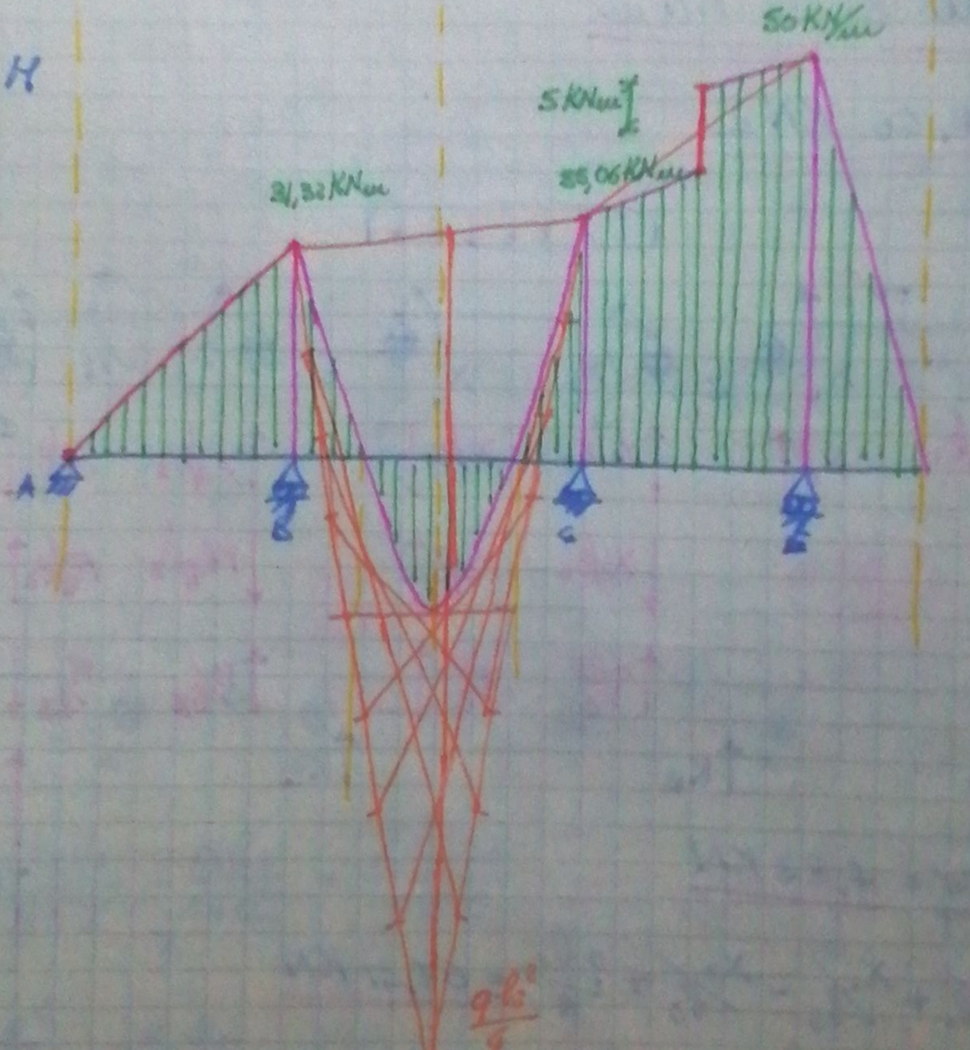
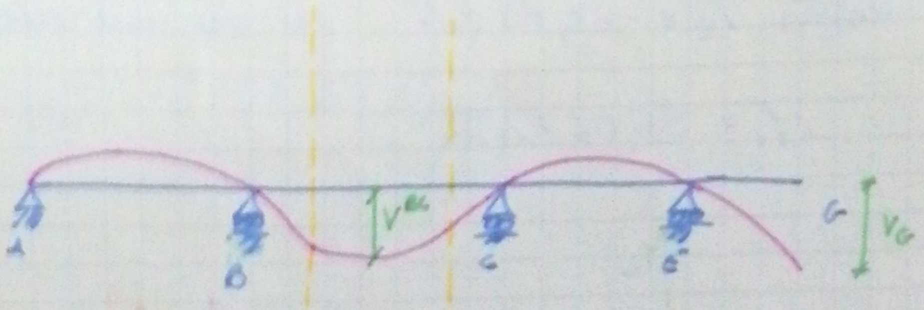


Diagrama M



Andamento linea elastica v



$$\theta + \theta^{t} = -\frac{d^2v}{dz^2}$$

$$\frac{d^2v}{dz^2} = -\frac{M}{EI} + \left(\frac{2\alpha\Delta t}{h}\right) \Rightarrow \begin{cases} \left.\dfrac{d^2v}{dz^2}\right|_{z=z_A} = \dfrac{2\alpha\Delta t}{h} > 0 \\[2ex] \left.\dfrac{d^2v}{dz^2}\right|_{z=z_B} = \dfrac{31,32\,kNm}{EI} + \dfrac{2\alpha\Delta t}{h} > 0 \end{cases}$$

La curvatura si mantiene positiva

$$v_{BC} = \frac{5}{384}\frac{q l_2^4}{EI} - \frac{X_1 l_2^2}{16EI} - \frac{X_2 l_2^2}{16EI} = 2,01\cdot10^{-2} - 4,19\cdot10^{-3} - 4,69\cdot10^{-3} =$$

$$= 0,01122\ m$$

$$v_{BC} = 11,22\ mm$$

$$\varphi_E = -\frac{X_2 l_3}{6EI} - \frac{M l_3}{24EI} - \frac{F l_4 l_3}{3EI} = -1,39\cdot10^{-3} - 9,92\cdot10^{-5} - 3,96\cdot10^{-3} =$$

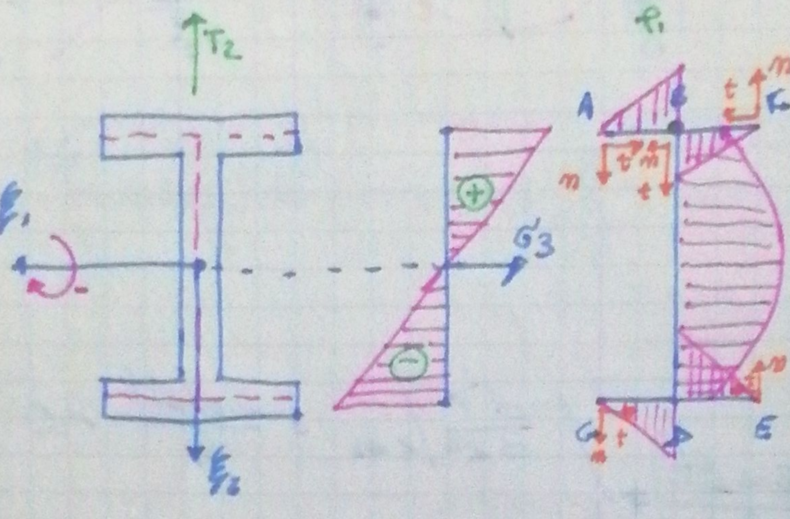
$$= -5,46\cdot10^{-3}$$

$$v_G = \varphi_E \cdot l_4 + \frac{F l_4^3}{3EI} = 0,0149\ m$$

$$v_G = 14,9\ mm$$

considerando la sezione C essa è sollecitata da un taglio $T_2 = -60{,}63\ KN$ e da un momento

$M_1 = 35{,}06\ KNm$



$$\sigma_3 = \frac{M_1}{I_1}\xi_2 \Rightarrow \sigma_3 = \frac{M_1}{W_x} = \frac{3506000\ Ncm}{533{,}27\ cm^3} = 6574{,}53\ N/cm^2$$

TRATTO AB

$$\tau_{t3}^{(P)} = -\frac{T_2 S_1'}{b_1 I_1} = -\frac{T_2}{\cancel{b_1} I_1}\cdot\left(\cancel{b_1}\cdot s\cdot\left(-\frac{h}{2}\right)\right) =$$

$$= -\frac{T_2}{I_1} s\left(-\frac{h}{2}\right)$$

$$\tau_{t3}^{(P_1)} = -\frac{T_2}{I_1}\left(-\frac{h}{2}\right)\cdot\frac{B}{2} = \frac{60{,}63\ KN}{7999\ cm^4}\cdot 7{,}5\ cm\cdot(-15\ cm) = 852{,}72\ N/cm^2$$

TRATTO BD

$$\tau_{t3}^{(P)} = -\frac{T_2 S_1'}{b_2 I_1} = -\frac{T_2}{b_2 I_1}\left[b_2\cdot s\cdot\left[-\left(\frac{H}{2}+\frac{s}{2}\right)\right] + b_1 B\cdot\left(-\frac{H}{2}\right)\right] =$$

$P_2\ (0, -139{,}3) \Rightarrow s = \frac{H}{2} - 139{,}3 = \frac{10{,}7}{2} - 5{,}37\ m$

$\tau_{t3}^{(P_2)} = -3071{,}42\ N/cm^2$

$\sigma_3^{(P_1)} = -6574,53 \ N/cm^2$

$\sigma_3^{(P_2)} = \frac{|M_1|}{I_1} \cdot 13,9.3 = -6105,59 \ N/cm^2$

Von Mises

$\sigma_{eq}^{(P_1)} = \sqrt{\sigma_3^{(P_1)2} + 3\,\tau_{t_3}^{(P_1)2}} = 6738,39 \ N/cm^2 < \sigma_{adm}$

$\sigma_{eq}^{(P_2)} = \sqrt{\sigma_3^{(P_2)2} + 3\,\tau_{t_3}^{(P_2)2}} = 8098,09 \ N/cm^2 < \sigma_{adm}$

Tresca

$\sigma_{eq}(P_1) = \sqrt{\sigma_3^2(P_1) + 4\,\tau_{t_3}^2(P_1)} = 6792,13 \ N/cm^2 < \sigma_{adm}$

$\sigma_{eq}(P_2) = \sqrt{\sigma_3^2(P_2) + 4\,\tau_{t_3}^2(P_2)} = 8660,99 \ N/cm^2 < \sigma_{adm}$