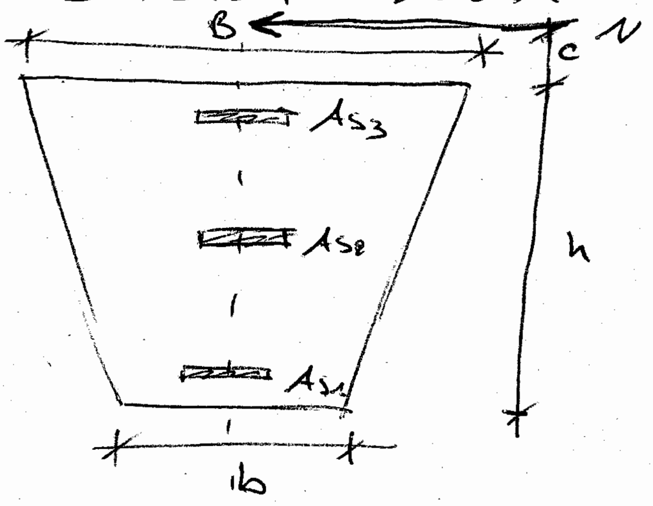
VERIFICA PRESSOFLESSIONE ALLE TENSIONI AMMISSIBILI DELLA SEZIONE: ①



$B = 40\,cm$; $b = 25\,cm$

$h = 45\,cm$; $d' = 3\,cm$

$A_{s1} = A_{s3} = 10{,}05\,cm^2$

$A_{s2} = 4{,}02\,cm^2$

$N = 300\,kN$

$c = 5\,cm$

CLS: $R_{ck} = 25\,MPa$; ACCIAIO: B450C (EX-FeB44k)

CHIARIMENTI SULL'ESERCIZIO:

NEL CASO DI PRESSOFLESSIONE RETTA A SEZIONE PARZIALIZZATA (GRANDE ECCENTRICITÀ), LA RICERCA DELL'ASSE NEUTRO, NECESSARIO PER PROCEDERE ALLA VERIFICA, SI RICONDUCE AL TROVARE IL PUNTO DI NULLO DELLA FUNZIONE:

$$\boxed{F(y) = S_n(y)(y+c) - I_n(y) = 0}$$

CHE, NEL CASO IN ESAME (SEZIONE A BASE VARIABILE LINEARMENTE LUNGO L'ALTEZZA) È UN'EQUAZIONE DI 4°.

SI ADOTTA, OVVIAMENTE, IL METODO ITERATIVO, PARTENDO DA $y_{c,1} = h$ (SEZIONE INTERAMENTE REAGENTE) ED ARRESTANDO LA RICERCA QUANDO, ALL'ITERAZIONE i-ESIMA: $\boxed{\Delta y_{c,i} < \frac{h}{1000}}$, IN

CUI: $$\boxed{\Delta y_{c,i} = \frac{F(y_{c,i})}{F'(y_{c,i})} = \frac{S_n(y_{c,i})(y_{c,i}+c) + I_n(y_{c,i})}{A_z(y_{c,i}+c) - S_n(y_{c,i})}}$$

I VALORI SUCCESSIVI DI y_c SI TROVANO CON LA:

$$\boxed{y_{c,i+1} = y_{c,i} - \Delta y_{c,i}}$$

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SVOLGIMENTO NUMERICO: $\boxed{y_{c1} = h = 45\,cm}$

$$A_z(y_{c,1}) = \frac{B+b}{2} h + n(A_{s1} + A_{s2} + A_{s3}) =$$

$$= \frac{40+25}{2} \cdot 45 + 15(10{,}05 + 4{,}02 + 10{,}05) = 1824{,}3\,cm$$

$$S_n(y_{c,1}) = \frac{B \cdot h^2}{2} - \left[\frac{(B-b)}{2} \cdot h \cdot \frac{h}{3}\right] + n\left[A_{s1} \cdot d' + A_{s2}\frac{h}{2} + A_{s3}(h-d')\right] =$$

$$= \frac{40 \cdot 45^2}{2} - \left[\frac{40-25}{2} \cdot \frac{45^2}{3}\right] + 15\left[10{,}05 \cdot 3 + 4{,}02 \cdot \frac{45}{2} + 10{,}05(45-3)\right] = 43578 \cdot cm^3$$

$$I_n(y_{c,1}) = \frac{B \cdot h^3}{3} - \frac{(B-b)h^3}{12} + n\left[A_{s1} d'^2 + A_{s2}\left(\frac{h}{2}\right)^2 + A_{s3}(h-d')^2\right] = \frac{40 \cdot 45^3}{3} - \frac{(40-25)^3}{12} +$$

$$+ 15\left[10{,}05 \cdot 3^2 + 4{,}02\left(\frac{45}{2}\right)^2 + 10{,}05(45-3)^2\right]$$

$$= 1512525{,}38\,cm^4$$

$$F'(y_{c,1}) = A_z(y_{c,1})(y_{c1} + c) - S_n(y_{c1});$$

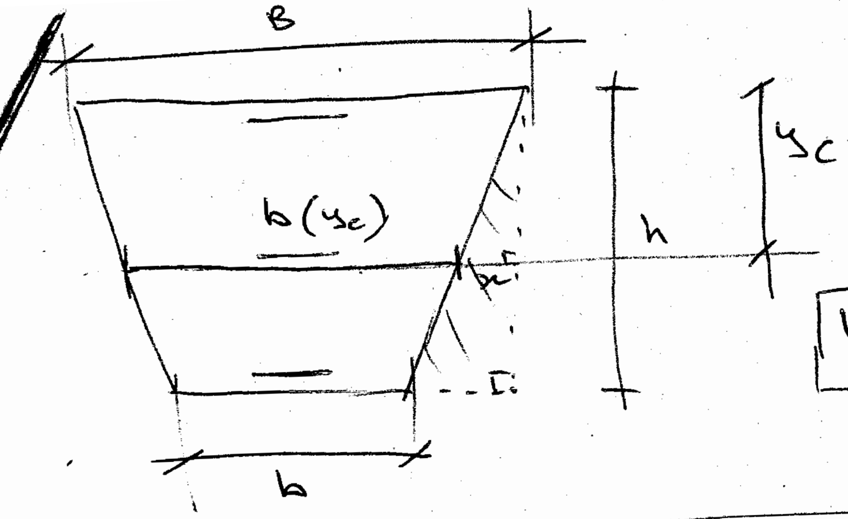
$$F(y_{c,1}) = S_n(y_{c,1})(y_{c1} + c) - I_n(y_{c,1});$$

$$F'(45) = 1824{,}3(45+5) - 43578 = 47637$$

$$F(45) = 43578(45+5) - 1512525{,}38 = 666374{,}62$$

$$\Delta y_{c,1} = \frac{F(y_{c1})}{F'(y_{c1})} = \frac{666374{,}62}{47637} = 13{,}98\,cm$$

$$\boxed{y_{c,2} = y_{c,1} - \Delta y_{c,1} = 45 - 13{,}98 = 31{,}02\,cm}$$



$$\frac{B-b}{2} : h = x : y_c$$

$$x = \frac{B-b}{2}\,\frac{y_c}{h}$$

$$\boxed{b(y_c) = B - (B-b)\frac{y_c}{h}}$$

$$b(y_{c2}) = 40-(40-25)\cdot\frac{31,02}{45} = 29,66\,cm$$

$$A_2(y_{c2}) = \frac{B+b(y_{c2})}{2}\,y_{c2} + n\,(A_{S1}+A_{S2}+A_{S3}) =$$

$$= \frac{40+29,66}{2}\cdot 31,02 + 15\,(2\cdot 10,05 + 4,02) = 1442,23\,cm^2$$

$$S_n(y_{c2}) = \frac{B\,y_{c2}^2}{2} - \frac{[B-b(y_{c2})]\,y_{c2}}{2}\cdot\frac{y_{c2}}{3} +$$

$$+ n\left[A_{S1}(y_{c2}-d) + A_{S2}\left(y_{c2}-\frac{h}{2}\right) + A_{S3}(y_{c2}-d')\right] =$$

$$= \frac{40\cdot 31,02^2}{2} - (40-29,66)\cdot\frac{31,02^2}{6} +$$

$$+ 15\left[10,05(31,02-42) + 4,02\left(31,02-\frac{45}{2}\right) + \right.$$

$$\left. + 10,05(31,02-3)\right] = 20669,08\,cm^3$$

$$I_n(y_{c2}) = \frac{B\,y_{c2}^3}{3} - [B-b(y_{c2})]\frac{y_{c2}^3}{12} + n\left[A_{S1}(y_{c2}-d)^2 + \right.$$

$$\left. + A_{S2}\left(y_{c2}-\frac{h}{2}\right)^2 + A_{S3}(y_{c2}-d')^2\right] =$$

$$= \frac{40\cdot 31,02^3}{3} - (40-29,66)\frac{31,02^3}{12} + 15\left[10,05(31,02-\right.$$

$$\left. + 4,02\left(31,02-\frac{45}{2}\right)^2 + 10,05(31,02-3)^2\right] =$$

$$= 513171,58\,cm^4$$

$$F'(y_{c2}) = 1442,23\,(31,02+5) - 20669,08 = 31280,04$$

$$F(y_{c2}) = 20669,08\,(31,02+5) - 513171,58 = 231328,68$$

(4)

$$\Delta y_{c2} = \frac{F(y_{c2})}{F'(y_{c2})} = \frac{231328{,}68}{31280{,}04} = 7{,}40\,cm$$

$$\boxed{y_{c3} = y_{c,2} - \Delta y_{c2} = 31{,}02 - 7{,}4 = 23{,}62\,cm}$$

$$b(y_{c3}) = 40 - (40-25)\frac{23{,}62}{45} = 32{,}13\,cm$$

$$A_z(y_{c3}) = \frac{40+32{,}13}{2}\cdot 23{,}62 + 15(2\cdot 10{,}05 + 4{,}02) = 1213{,}65c$$

$$S_n(y_{c3}) = \frac{40\cdot 23{,}62^2}{2} - \frac{40-32{,}13}{2}\cdot\frac{23{,}62^2}{3} +$$
$$+ 15\left[10{,}05(23{,}62-42) + 4{,}02\left(23{,}62 - \frac{45}{2}\right) +\right.$$
$$\left.+ 10{,}05(23{,}62-3)\right] = 10831{,}52\,cm^3$$

$$I_n(y_{c3}) = \frac{40\cdot 23{,}62^3}{3} - (40-32{,}13)\frac{23{,}62^3}{12} + 15\left[10{,}05(23{,}62-42\right.$$
$$\left.+ 4{,}02\left(23{,}62-\frac{45}{2}\right)^2 + 10{,}05(23{,}62-3)^2\right] = 282160\,cm^4$$

$$F(23{,}62) = 10831{,}52(23{,}62+5) - 282160 = 27838{,}10$$

$$F'(23{,}62) = 1213{,}65(23{,}62+5) - 10831{,}52 = 23903{,}14$$

$$\Delta y_{c3} = \frac{F(y_{c3})}{F'(y_{c3})} = \frac{27838{,}10}{23903{,}14} = 1{,}16\,cm$$

$$\boxed{y_{c4} = y_{c3} - \Delta y_{c3} = 23{,}62 - 1{,}16 = 22{,}45\,cm}$$

$$b(y_{c4}) = 40 - (40-25)\frac{22{,}45}{45} = 32{,}52\,cm$$

$$A_z(y_{c4}) = \frac{40+32{,}52}{2}\,22{,}45 + 15(2\cdot 10{,}05 + 4{,}02) = 1175{,}84\,cm^2$$

$$S_n(y_{c4}) = \frac{40\cdot 22{,}45^2}{2} - \frac{40-32{,}52}{2}\cdot\frac{22{,}45^2}{3} +$$
$$+ 15\left[10{,}05(22{,}45-42) + 4{,}02\left(22{,}45-\frac{45}{2}\right) +\right.$$
$$\left.+ 10{,}05(22{,}45-3)\right] = 9433{,}64\,cm^3$$

$$I_n(y_{c4}) = \frac{40\cdot 22{,}45^3}{3} - (40-32{,}52)\cdot\frac{22{,}45^3}{12} +$$
$$+ 15\left[10{,}05(22{,}45-42)^2 + 4{,}02\left(22{,}45-\frac{45}{2}\right)^2 +\right.$$
$$\left.10{,}05(22{,}45-3)^2\right] = 258458\,cm^4$$

$F(22{,}45) = 9433{,}64\,(22{,}45+5) - 258458 = 495{,}42$ (5)

$F'(22{,}45) = 1175{,}84\,(22{,}45+5) - 9433{,}64 = 22843{,}168$

$$\Delta y_{c4} = \frac{495{,}42}{22843{,}168} = 2{,}169 \cdot 10^{-2}\ \mathrm{cm}$$

$$\frac{h}{1000} = \frac{45}{1000} = 0{,}045 > 0{,}02169 \qquad \boxed{y_c = 22{,}45\ \mathrm{cm}}$$

VERIFICHE

$$\sigma_c = \frac{N}{S_n} \cdot y_c = \frac{300 \cdot 1000}{9433{,}64 \cdot 1000} \cdot 224{,}5 = \left[\begin{array}{c}\text{ESPRIMO TUTTO IN} \\ N \text{ E } mm\end{array}\right] =$$

$$= 7{,}13\ \mathrm{MPa} < \bar{\sigma}_c = 8{,}5\ \mathrm{MPa}$$

$$\sigma_{s1} = n\,\frac{N}{S_n}\,(d - y_c) = 15 \cdot \frac{300 \cdot 1000}{9433{,}64 \cdot 1000}\,(420 - 22{,}45) =$$

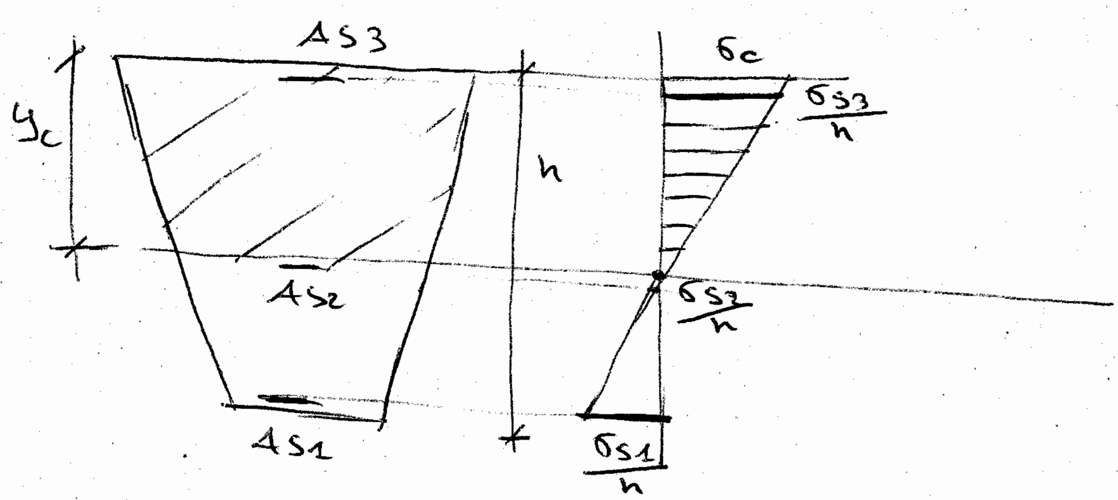
$$= 93{,}23\ \mathrm{MPa} < \bar{\sigma}_s = 260\ \mathrm{MPa}$$

$$\sigma_{s2} = n\,\frac{N}{S_n}\,\left(\frac{h}{2} - y_c\right) = 15 \cdot \frac{300 \cdot 1000}{9433{,}64 \cdot 1000}\,(225 - 224{,}5)$$

$$= 0{,}239\ \mathrm{MPa}$$

$$\sigma_{s3} = n\,\frac{N}{S_n}\,(y_c - d') = 15 \cdot \frac{300 \cdot 1000}{9433{,}64 \cdot 1000}\,(224{,}5 - 30) =$$

$$= 92{,}78\ \mathrm{MPa}$$

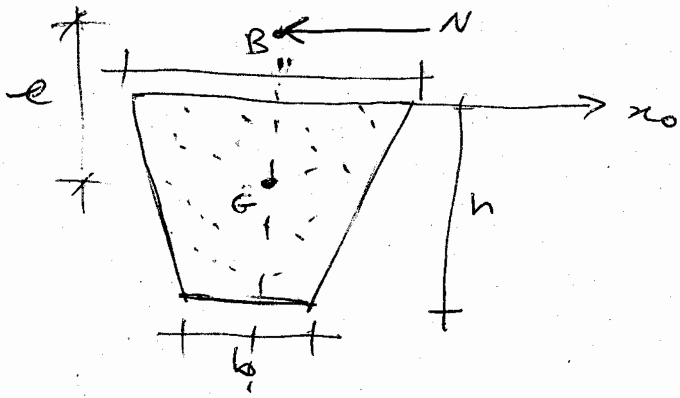


N. B. NEL CASO LA TRACCIA FORNISCA
IL VALORE DI e (ECCENTRICITÀ DEL CENTRO DI PRESSIONE RISPETTO AL BARICENTRO), VA APPLICATA LA RELAZIONE

$$C = e - x_G$$

DOVE x_G È LA POSIZIONE DI G (SEZIONE DI SOLO CLS), CALCOLABILE CON:

$$x_G = \frac{S_{x_0}}{A}$$



NEL CASO IN ESAME:

$$S_{x_0} = \frac{bh^2}{2} + \frac{(B-b)}{2} h \cdot \frac{h}{3} = \frac{25 \cdot 45^2}{2} + \frac{(40-25)}{2} \cdot \frac{45^2}{3} =$$
$$= 30375 \, cm^3$$

$$A = (B+b)\frac{h}{2} = 1462{,}5 \, cm^2$$

$$\boxed{x_G = \frac{30375}{1462{,}5} = 20{,}77 \, cm}$$