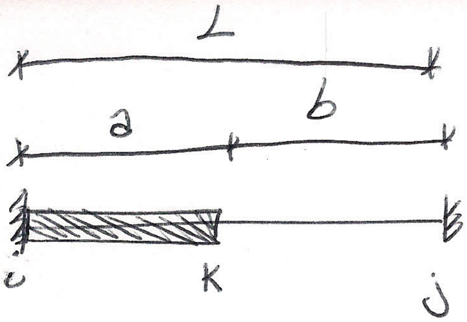
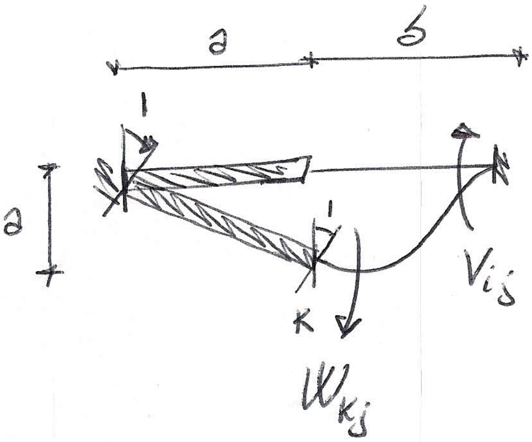
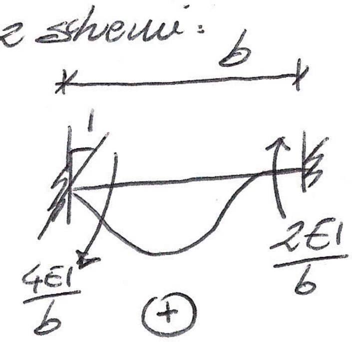


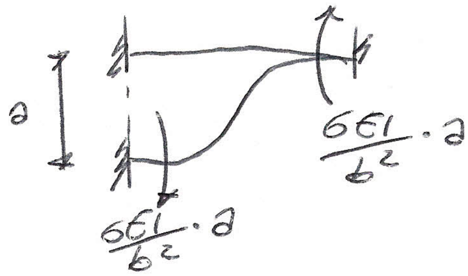
Calcolo dei Coefficienti di Rigidezza dell'asta:

$\varphi_i = 1$ → Calcolo W_{ij}, V_{ij}



Si può vedere la parte deformabile come sovrapposizione di 2 schemi:

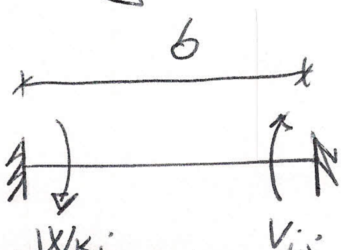




$$W_{kj} = \frac{4EI}{b} + \frac{6EI}{b^2} a$$

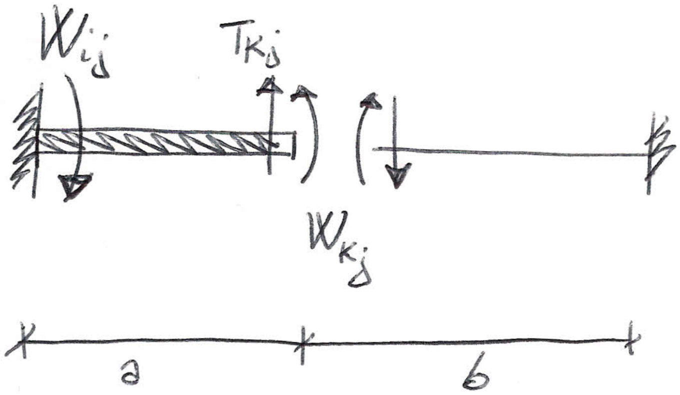
$$\boxed{V_{ij} = \frac{2EI}{b} + \frac{6EI}{b^2} \cdot a}$$

Considerando i tagli che si generano da W_{kj} e V_{ij}



↓ $T_{kj} = \dfrac{W_{kj} + V_{ij}}{b}$ ↑ $T_{kj} = \dfrac{W_{kj} + V_{ij}}{b}$

Sull'estremo K si ha:

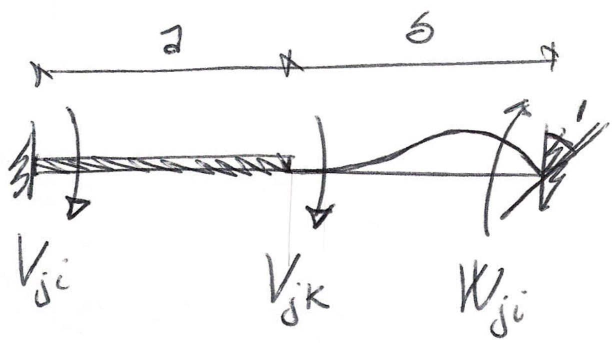


Per equilibrio si determina W_{ij}:

$$\boxed{W_{ij} = W_{kj} + T_{kj} \cdot a}$$

Applicando una rotazione unitaria in j si determinano:

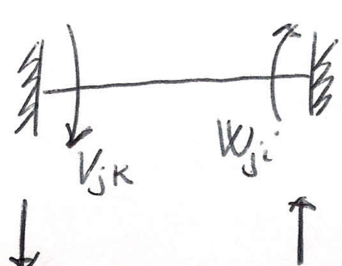
$\varphi_j = 1 \rightarrow$ Calcolo V_{ji}, W_{ji}



$$\boxed{W_{ji} = \frac{4EI}{b}}$$

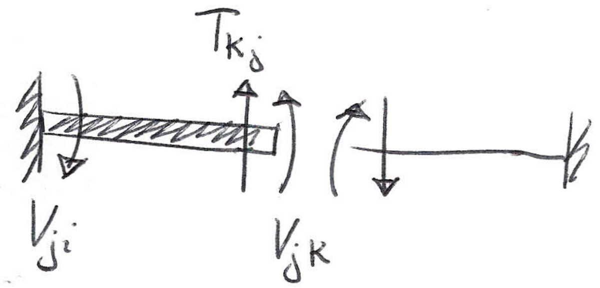
$$V_{jK} = \frac{2EI}{b}$$

Considerando i tagli che nascono per effetto di V_{jK} e W_{ji}



$$T_{Kj} = \frac{V_{jK} + W_{ji}}{b}$$

V_{ji} può essere determinato come prima per eq. alla rotazione:



$$\boxed{V_{ji} = V_{jk} + T_{kj} \cdot a}$$

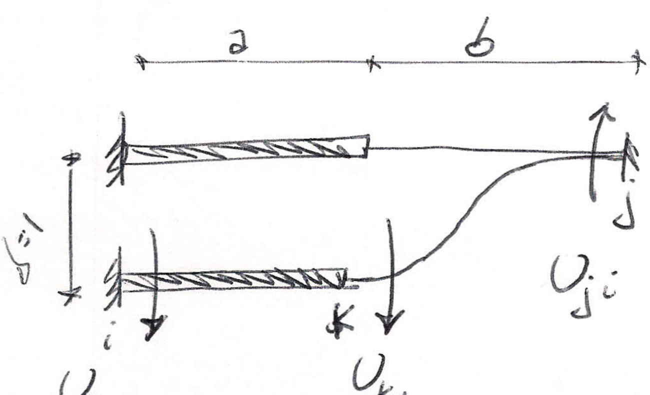
Coeff. di rigidezza per traslazione unitaria:

Come noto possono essere determinati come:

$$U_{ij} = \frac{W_{ij} + V_{ij}}{a+b}$$

$$U_{ji} = \frac{W_{ji} + V_{ji}}{a+b}$$

Volendoli ricavare come fatto prima si consideri:

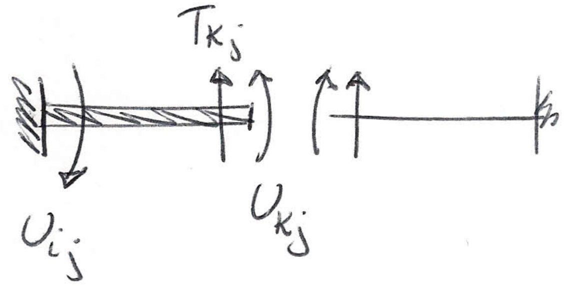


$$\boxed{U_{ji} = \frac{6EI}{b^2}}$$

$$U_{kj} = \frac{6EI}{b^2}$$

$$T_{kj} = \frac{U_{ji} + U_{kj}}{b}$$

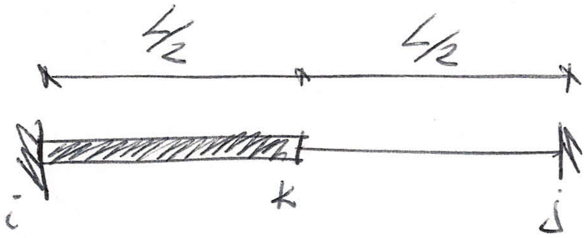
Per equilibrio:



$$\boxed{V_{ij} = V_{kj} + T_{kj} \cdot a}$$

* Si fa notare come applicando lo spostamento unitario all'estremo j si ottengono gli stessi valori di V_{ij} e V_{ji}.

<u>Esempio simbolico con $a = b = \frac{L}{2}$</u>



$\varphi_i = 1$

$$W_{kj} = \frac{4EI}{L/2} + \frac{6EI}{(L/2)^2} \cdot \frac{L}{2} = \frac{8EI}{L} + \frac{12EI}{L} = \frac{20EI}{L}$$

$$V_{ij} = \frac{2EI}{L/2} + \frac{6EI}{(L/2)^2} \cdot \frac{L}{2} = \frac{4EI}{L} + \frac{12EI}{L} = \frac{16EI}{L}$$

$$T_{kj} = \frac{W_{kj} + V_{ij}}{L/2} = \frac{36EI}{L} \cdot \frac{2}{L} = \frac{72EI}{L^2}$$

$$W_{ij} = \frac{20EI}{L} + \frac{72EI}{L^2} \cdot \frac{L}{2} = \frac{56EI}{L}$$

$\varphi_j = 1$

$$\underline{\underline{W_{ji}}} = \frac{4EI}{L/2} = \underline{\underline{\frac{8EI}{L}}}$$

$$V_{jk} = \frac{2EI}{L/2} = \frac{4EI}{L}$$

$$T_{kj} = \frac{V_{jk} + W_{ji}}{L/2} = \frac{12EI}{L} \cdot \frac{2}{L} = \frac{24EI}{L^2}$$

$$\underline{\underline{V_{ji}}} = V_{jk} + T_{kj} \cdot \frac{L}{2} = \frac{4EI}{L} + \frac{24EI}{L^{\cancel{2}}} \cdot \frac{\cancel{L}}{2} = \underline{\underline{\frac{16EI}{L}}}$$

$\delta = 1$

$$\underline{\underline{U_{ij}}} = \frac{W_{ij} + V_{ij}}{L} = \left[\frac{56EI}{L} + \frac{16EI}{L}\right] \cdot \frac{1}{L} = \underline{\underline{\frac{72EI}{L^2}}}$$

$$\underline{\underline{U_{ji}}} = \frac{W_{ji} + V_{ji}}{L} = \left[\frac{8EI}{L} + \frac{16EI}{L}\right] \cdot \frac{1}{L} = \underline{\underline{\frac{24EI}{L^2}}}$$

Per verifica volendoli determinare dalle relazioni trovate a partire dalla deformata.

$$\underline{\underline{U_{ji}}} = \frac{6EI}{(L/2)^2} = \underline{\underline{\frac{24EI}{L^2}}} \qquad \text{OK!}$$

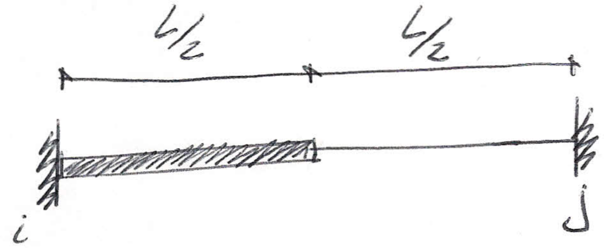
$$U_{kj} = \frac{6EI}{\left(\frac{L}{2}\right)^2} = \frac{24EI}{L^2}$$

$$T_{kj} = \frac{U_{jk} + U_{kj}}{\frac{L}{2}} = \frac{48EI}{L^2} \cdot \frac{2}{L} = \frac{96EI}{L^3}$$

$$\underline{\underline{U_{ij}}} = U_{kj} + T_{kj} \cdot \frac{L}{2} = \frac{24EI}{L^2} + \frac{\cancel{96}^{\,48}EI}{L^{\cancel{3}\,2}} \cdot \frac{\cancel{L}}{\cancel{2}} = \underline{\underline{\frac{72EI}{L^2}}}$$

ok!

<u>Ricapitolando:</u>



$$W_{ij} = \frac{56EI}{L} \qquad W_{ji} = \frac{8EI}{L}$$

$$V_{ij} = \frac{16EI}{L} \qquad V_{ji} = \frac{16EI}{L}$$

$$U_{ij} = \frac{72EI}{L^2} \qquad U_{ji} = \frac{24EI}{L^2}$$

$$\mu_{ij} = -\frac{qL^2}{12} \qquad \mu_{ji} = \frac{qL^2}{12}$$