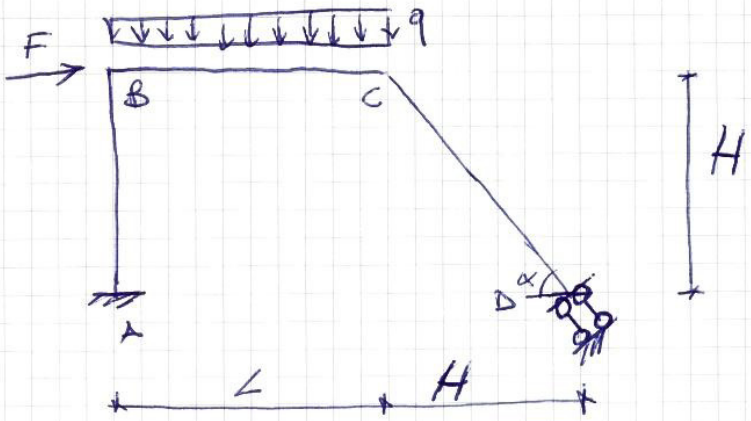
Esercizio - Asta inclinata con doppio pendolo parallelo



$L = 5000\ mm$
$H = 3000\ mm$
$q = 40\ N/mm$
$F = 2 \cdot 10^4\ N$

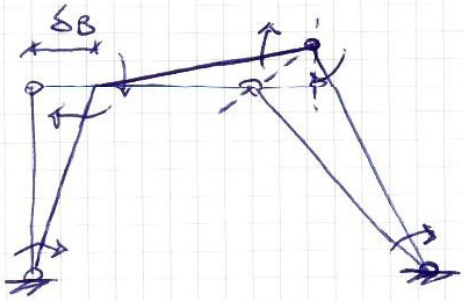
ritti
$b_r = 300\ mm$
$h_r = 500\ mm$
$I_r = 3{,}125 \cdot 10^9\ mm^4$

trave
$b_t = 300\ mm$
$h_t = 600\ mm$
$I_t = 5{,}4 \cdot 10^9\ mm^4$

$f_{ck} = 20\ MPa \Rightarrow E = 22000 \left(\frac{20+8}{10}\right)^{0,3} = 29900\ MPa$

Dalla struttura reticolare associata si ha:

$$3t - 2c = 3 \cdot 3 - 2 \cdot 4 = 1$$



incognite $\{\varphi_B, \varphi_C, \delta_B\}$

	δ'_B	δ_B
δ_{AB}	1	1
δ_{BC}	$-\frac{1}{tg\alpha}$	$-\frac{1}{tg\alpha}$
δ_{CD}	$\frac{1}{\sin\alpha}$	$\frac{1}{\sin\alpha}$

$\alpha = 45° \Rightarrow \frac{1}{tg\alpha} = 1$

$\frac{1}{\sin\alpha} = \sqrt{2}$

Equazioni risolutive:

$$\begin{cases} 1)\ M_{BA} + M_{BC} = 0 \\ 2)\ M_{CB} + M_{CA} = 0 \\ 3)\ \overset{TLV}{\Rightarrow} \dfrac{M_{AB}+M_{BA}}{H}\cdot\delta'_{AB} + \dfrac{M_{BC}+M_{CB}}{L}\delta'_{BC} + \dfrac{M_{CD}+M_{DC}}{\sqrt{2}\,H}\delta'_{CA} + F\delta'_B + \\ \quad - \dfrac{qL}{2}\cdot\delta'_B = 0 \end{cases}$$

$$\begin{cases} M_{BA} + M_{BC} = 0 \\ M_{CB} + M_{CA} = 0 \\ \dfrac{M_{AB}+M_{BA}}{H}\delta'_B - \dfrac{M_{BC}+M_{CB}}{L}\delta'_B + \dfrac{M_{CA}+M_{DC}}{\sqrt{2}\,H}\cdot\sqrt{2}\,\delta'_B + F\delta'_B - \dfrac{qL}{2}\delta'_B = 0 \end{cases}$$

Risoluzione Numerica:

Asta AB

$$W_{AB} = W_{BA} = \frac{4EI}{H} = 1{,}25\cdot 10^{11}\ N$$

$$V_{AB} = V_{BA} = \frac{2EI}{H} = 6{,}23\cdot 10^{10}\ N$$

$$U_{AB} = U_{BA} = \frac{6EI}{H^2} = 6{,}23\cdot 10^{7}\ N$$

Asta BC

$$W_{BC} = W_{CB} = \frac{4EI}{L} = 1{,}29\cdot 10^{11}\ N$$

$$V_{BC} = V_{CB} = \frac{2EI}{L} = 6{,}46\cdot 10^{10}\ N$$

$$U_{BC} = U_{CB} = \frac{6EI}{L^2} = 3{,}87\cdot 10^{7}\ N$$

$$M_{BC} = -M_{CB} = -\frac{qL^2}{12} = -8{,}33\cdot 10^{7}\ N$$

Asta CD

$$W_{CD} = W_{DC} = \frac{EI}{\sqrt{2}H} = 2{,}20 \cdot 10^{10}\ N$$

$$V_{CD} = V_{DC} = -\frac{EI}{\sqrt{2}H} = 2{,}20 \cdot 10^{10}\ N$$

$$U_{CD} = U_{DC} = 0$$

$$\begin{cases} M_{AB} = 6{,}23 \cdot 10^{10}\varphi_B - 6{,}23 \cdot 10^{7}\delta_B \\ M_{BA} = 1{,}25 \cdot 10^{11}\varphi_B - 6{,}23 \cdot 10^{7}\delta_B \\ M_{BC} = 1{,}29 \cdot 10^{11}\varphi_B + 6{,}46 \cdot 10^{10}\varphi_C + 3{,}87 \cdot 10^{7}\delta_B - 8{,}33 \cdot 10^{7} \\ M_{CB} = 1{,}29 \cdot 10^{11}\varphi_C + 6{,}46 \cdot 10^{10}\varphi_B + 3{,}87 \cdot 10^{7}\delta_B + 8{,}33 \cdot 10^{7} \\ M_{CD} = 2{,}20 \cdot 10^{10}\varphi_C \\ M_{DC} = -2{,}20 \cdot 10^{10}\varphi_C \end{cases}$$

$$\begin{cases} \left(1{,}25 \cdot 10^{11} + 1{,}29 \cdot 10^{11}\right)\varphi_B + 6{,}46 \cdot 10^{10}\varphi_C + \left(3{,}87 \cdot 10^{7} - 6{,}23 \cdot 10^{7}\right)\delta_B = 8{,}33 \cdot 10^{7} \\ 6{,}46 \cdot 10^{10}\varphi_B + \left(1{,}29 \cdot 10^{11} + 2{,}20 \cdot 10^{10}\right)\varphi_C + 3{,}87 \cdot 10^{7}\delta_B = -8{,}33 \cdot 10^{7} \\ \dfrac{\left(6{,}23 \cdot 10^{10} + 1{,}25 \cdot 10^{11}\right)\varphi_B - \left(6{,}23 \cdot 10^{7} \cdot 2\right)\delta_B}{3000} - \dfrac{\left(1{,}29 \cdot 10^{11} + 6{,}46 \cdot 10^{10}\right)\varphi_B + \left(1{,}29 \cdot 10^{11} + 6{,}46 \cdot 10^{10}\right)\varphi_C + \left(3{,}87 \cdot 10^{7} \cdot 2\right)\delta_B}{5000} + \dfrac{\left(2{,}20 \cdot 10^{10} - 2{,}20 \cdot 10^{10}\right)\varphi_C}{3000} + 2 \cdot 10^{4} - \dfrac{40 \cdot 5000}{2} = 0 \end{cases}$$

$$\begin{cases} 2{,}54\cdot 10^{11}\varphi_B + 6{,}46\cdot 10^{10}\varphi_C - 2{,}36\cdot 10^{7}\delta_B = 8{,}33\cdot 10^{7} \\ 6{,}46\cdot 10^{10}\varphi_B + 1{,}51\cdot 10^{11}\varphi_C + 3{,}87\cdot 10^{7}\delta_B = -8{,}33\cdot 10^{7} \\ 2{,}36\cdot 10^{7}\varphi_B - 3{,}87\cdot 10^{7}\varphi_C - 5{,}7\cdot 10^{4}\delta_B = 8\cdot 10^{4} \end{cases}$$

$$\begin{bmatrix} 2{,}54\cdot 10^{11} & 6{,}46\cdot 10^{10} & -2{,}36\cdot 10^{7} \\ 6{,}46\cdot 10^{10} & 1{,}51\cdot 10^{11} & 3{,}87\cdot 10^{7} \\ 2{,}36\cdot 10^{7} & -3{,}87\cdot 10^{7} & -5{,}7\cdot 10^{4} \end{bmatrix} \begin{bmatrix} \varphi_B \\ \varphi_C \\ \delta_B \end{bmatrix} = \begin{bmatrix} 8{,}33\cdot 10^{7} \\ -8{,}33\cdot 10^{7} \\ 8\cdot 10^{4} \end{bmatrix}$$

$$\Downarrow$$

$$\begin{cases} \varphi_B = 3{,}59\cdot 10^{-4} \\ \varphi_C = -4{,}64\cdot 10^{-4} \\ \delta_B = -0{,}9398 \text{ mm} \end{cases}$$

Momenti:

$M_{AB} = 80{,}92$ KNm

$M_{BA} = 103{,}42$ KNm

$M_{BC} = -103{,}42$ KNm

$M_{CB} = 102{,}08$ KNm

$M_{CD} = -102{,}08$ KNm

$M_{DC} = 10{,}208$ KNm

Tagli:

$$T_{AB} = T_{BA} = -\frac{M_{AB}+M_{BA}}{H} = -61{,}45 \text{ KN}$$

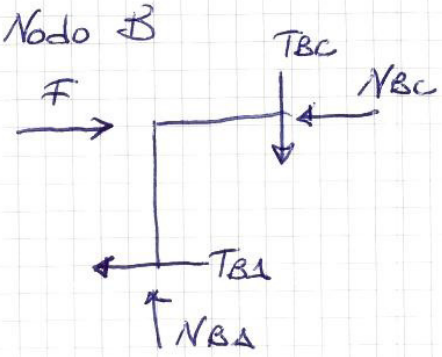
$$T_{BC} = \frac{qL}{2} - \frac{M_{BC}+M_{CB}}{L} = 118{,}64 \text{ KN}$$

$$T_{CB} = -\frac{qL}{2} - \frac{M_{BC}+M_{CB}}{L} = -81{,}36 \text{ KN}$$

$$T_{CD} = T_{DC} = 0$$

Sforzi Normali

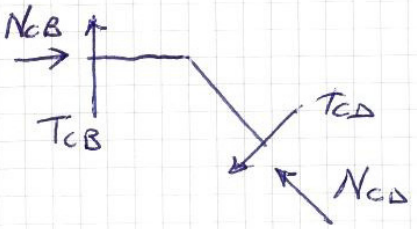
Nodo B



$N_{BA} = T_{BC} = 118,66\ kN$

$N_{BC} = F - T_{BA} = 20 + 61,65 = 81,65\ kN$

Nodo C

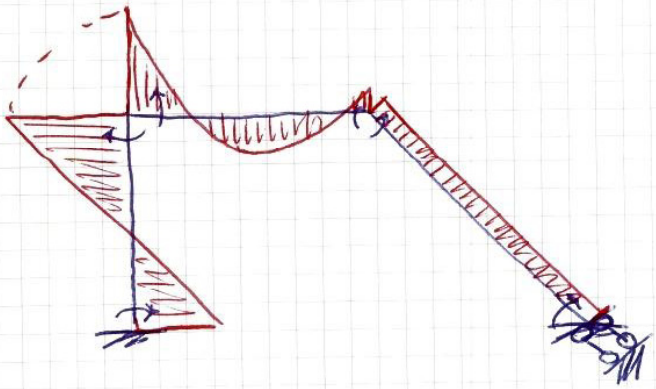


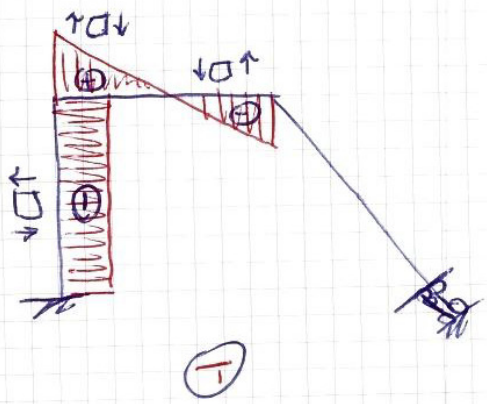
$$\begin{cases} N_{CB} - \frac{T_{CA}}{\sqrt{2}} - \frac{N_{CA}}{\sqrt{2}} = 0 \\ T_{CB} - \frac{T_{CA}}{\sqrt{2}} + \frac{N_{CA}}{\sqrt{2}} = 0 \end{cases}$$

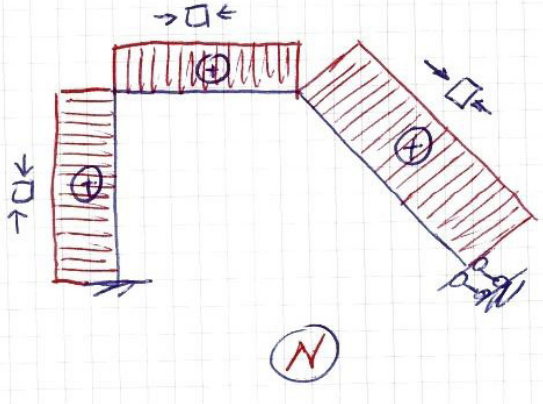
$$\Downarrow$$

$$N_{CA} = \left(\frac{T_{CA}}{\sqrt{2}} - T_{CB}\right)\sqrt{2} = 115,06\ kN$$

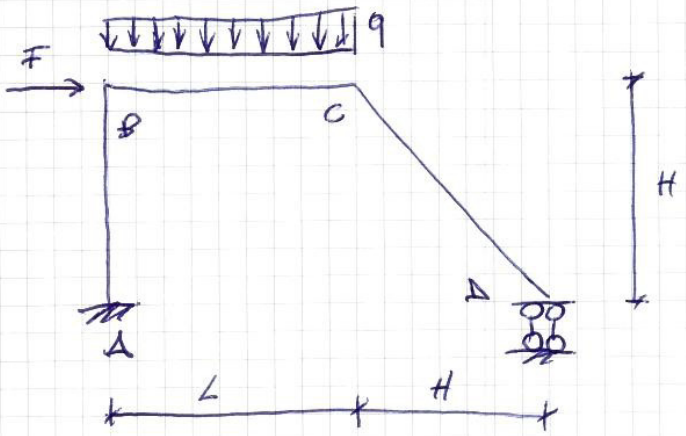
$$N_{CB} = \frac{T_{CA}}{\sqrt{2}} + \frac{N_{CA}}{\sqrt{2}} = 81,36\ kN \quad (OK)$$







Esercizio: Asta inclinata e pendolo verticale



$L = 5000$ mm
$H = 3000$ mm
$q = 40$ N/mm
$F = 2 \cdot 10^4$ N
$E = 29900$ MPa

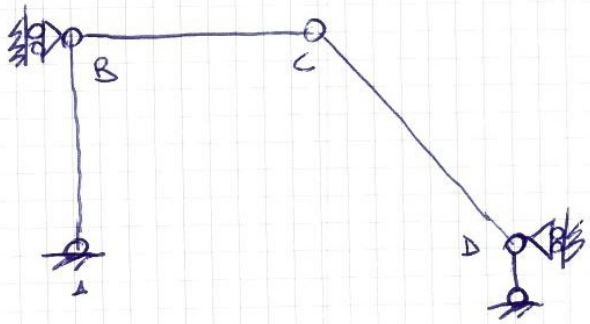
Ritti
$b = 300$ mm
$h = 500$ mm
$I = 3{,}125 \cdot 10^9$ mm^4

Trave
$b = 300$ mm
$h = 600$ mm
$I = 5{,}4 \cdot 10^9$ mm^4

N.B. → Lo spostamento causato dal doppio pendolo non è ortogonale all'asta quindi è un ulteriore incognita del problema

↓

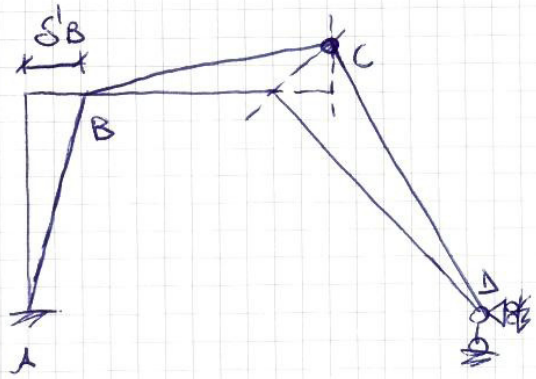
telaio a 2 nodi spostabili



incognite

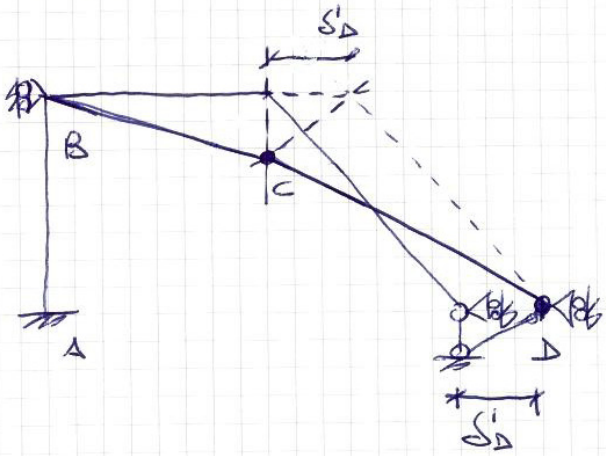
$\{\varphi_B, \varphi_C, \delta_B, \delta_D\}$

Cinematismo ①



	δ'_B
δ_{AB}	1
δ_{BC}	-1
δ_{CD}	$\sqrt{2}$

Cinematismo ②



	δ'_D
δ_{AB}	0
δ_{BC}	1
δ_{CD}	$-\sqrt{2}$

Spostamenti reali:

$$\begin{cases} \delta_{AB} = \delta_B \\ \delta_{BC} = -\delta_B + \delta_D \\ \delta_{CD} = \sqrt{2}\,\delta_B - \sqrt{2}\,\delta_D \end{cases}$$

Eq. risolutive:

$$\begin{cases} M_{BA} + M_{BC} = 0 \\ M_{CB} + M_{CD} = 0 \\ PLV(1)\ \dfrac{M_{AB}+M_{BA}}{H}\delta_B - \dfrac{M_{BC}+M_{CB}}{L}\delta'_B + \dfrac{M_{CD}+M_{DC}}{\sqrt{2}H}\sqrt{2}\,\delta'_B + F\delta'_B - \dfrac{qL\delta'_B}{2} = 0 \\ PLV(2)\ \dfrac{M_{BC}+M_{CB}}{L}\delta'_D - \dfrac{M_{CD}+M_{DC}}{\sqrt{2}H}\sqrt{2}\,\delta'_D + qL\dfrac{\delta'_D}{2} = 0 \end{cases}$$

Risoluzione Numerica

Asta AB

$$W_{AB} = W_{BA} = \frac{4EI}{H} = 1{,}25 \cdot 10^{11}\ Nmm$$

$$V_{AB} = V_{BA} = \frac{2EI}{H} = 6{,}23 \cdot 10^{10}\ Nmm$$

$$U_{AB} = U_{BA} = \frac{6EI}{H^2} = 6{,}23 \cdot 10^{7}\ N$$

Asta BC

$$W_{BC} = W_{CB} = \frac{4EI}{L} = 1{,}29 \cdot 10^{11}\ Nmm$$

$$V_{BC} = V_{CB} = \frac{2EI}{L} = 6{,}46 \cdot 10^{10}\ Nmm$$

$$U_{BC} = U_{CB} = \frac{6EI}{L^2} = 3{,}87 \cdot 10^{7}\ N$$

$$M_{BC} = -M_{CB} = -\frac{qL^2}{12} = -8{,}33 \cdot 10^{7}\ Nmm$$

Asta CD

$$W_{CD} = W_{DC} = \frac{4EI}{\sqrt{2}H} = 8{,}81 \cdot 10^{10}\ Nmm$$

$$V_{CD} = V_{DC} = \frac{2EI}{\sqrt{2}H} = 4{,}40 \cdot 10^{10}\ Nmm$$

$$U_{CD} = U_{DC} = \frac{6EI}{(\sqrt{2}H)^2} = 3{,}11 \cdot 10^{7}\ N$$

$$\begin{cases} M_{AB} = 6{,}23 \cdot 10^{10}\,\varphi_B - 6{,}23 \cdot 10^{7} \cdot \delta_B \\ M_{BA} = 1{,}25 \cdot 10^{11}\,\varphi_B - 6{,}23 \cdot 10^{7}\,\delta_B \\ M_{BC} = 1{,}29 \cdot 10^{11}\,\varphi_B + 6{,}46 \cdot 10^{10}\,\varphi_C + 3{,}87 \cdot 10^{7}\,\delta_B - 3{,}87 \cdot 10^{7}\,\delta_D - 8{,}33 \cdot 10^{7} \\ M_{CB} = 6{,}46 \cdot 10^{10}\,\varphi_B + 1{,}29 \cdot 10^{11}\,\varphi_C + 3{,}87 \cdot 10^{7}\,\delta_B - 3{,}87 \cdot 10^{7}\,\delta_D + 8{,}33 \cdot 10^{7} \\ M_{CD} = 8{,}81 \cdot 10^{10}\,\varphi_C - 3{,}11 \cdot 10^{7} \cdot \sqrt{2} \cdot \delta_B + 3{,}11 \cdot 10^{7} \cdot \sqrt{2} \cdot \delta_D \\ M_{DC} = 4{,}40 \cdot 10^{10}\,\varphi_C - 3{,}11 \cdot 10^{7} \cdot \sqrt{2}\,\delta_B + 3{,}11 \cdot 10^{7} \cdot \sqrt{2} \cdot \delta_D \end{cases}$$

$$\begin{cases} 2{,}54\cdot10^{11}\varphi_B + 6{,}46\cdot10^{10}\varphi_C - 2{,}36\cdot10^{7}\delta_B - 3{,}87\cdot10^{7}\delta_D = 8{,}33\cdot10^{7} \\ 6{,}46\cdot10^{10}\varphi_B + 2{,}17\cdot10^{11}\varphi_C - 5{,}28\cdot10^{6}\delta_B + 5{,}28\cdot10^{6}\delta_D = -8{,}33\cdot10^{7} \\ 6{,}24\cdot10^{7}\varphi_B - 4{,}15\cdot10^{4}\delta_B - 3{,}87\cdot10^{7}\varphi_B - 3{,}87\cdot10^{7}\varphi_C - 1{,}55\cdot10^{4}\delta_B + \\ \quad + 1{,}55\cdot10^{4}\delta_D + 4{,}40\cdot10^{7}\varphi_C - 2{,}93\cdot10^{4}\delta_B + 2{,}93\cdot10^{4}\delta_D + \\ \quad 2\cdot10^{4} - 1{,}0\cdot10^{5} = 0 \\ 3{,}87\cdot10^{7}\varphi_B + 3{,}87\cdot10^{7}\varphi_C + 1{,}55\cdot10^{4}\delta_B - 1{,}55\cdot10^{4}\delta_D - 4{,}40\cdot10^{7}\varphi_C + \\ \quad + 2{,}93\cdot10^{4}\delta_B - 2{,}93\cdot10^{4}\delta_D + 1{,}0\cdot10^{5} = 0 \end{cases}$$

$$\begin{cases} 2{,}54\cdot10^{11}\varphi_B + 6{,}46\cdot10^{10}\varphi_C - 2{,}36\cdot10^{7}\delta_B - 3{,}87\cdot10^{7}\delta_D = 8{,}33\cdot10^{7} \\ 6{,}46\cdot10^{10}\varphi_B + 2{,}17\cdot10^{11}\varphi_C - 5{,}28\cdot10^{6}\delta_B + 5{,}28\cdot10^{6}\delta_D = -8{,}33\cdot10^{7} \\ 2{,}36\cdot10^{7}\varphi_B + 5{,}28\cdot10^{6}\varphi_C - 8{,}58\cdot10^{4}\delta_B + 4{,}48\cdot10^{4}\delta_D = 8\cdot10^{4} \\ 3{,}87\cdot10^{7}\varphi_B - 5{,}28\cdot10^{6}\varphi_C + 4{,}48\cdot10^{4}\delta_B - 4{,}48\cdot10^{4}\delta_D = -1{,}0\cdot10^{5} \end{cases}$$

$$\begin{bmatrix} 2{,}54\cdot10^{11} & 6{,}46\cdot10^{10} & -2{,}36\cdot10^{7} & -3{,}87\cdot10^{7} \\ 6{,}46\cdot10^{10} & 2{,}17\cdot10^{11} & -5{,}28\cdot10^{6} & 5{,}28\cdot10^{6} \\ 2{,}36\cdot10^{7} & 5{,}28\cdot10^{6} & -8{,}58\cdot10^{4} & 4{,}48\cdot10^{4} \\ 3{,}87\cdot10^{7} & -5{,}28\cdot10^{6} & 4{,}48\cdot10^{4} & -4{,}48\cdot10^{4} \end{bmatrix} \begin{bmatrix} \varphi_B \\ \varphi_C \\ \delta_B \\ \delta_D \end{bmatrix} = \begin{bmatrix} 8{,}33\cdot10^{7} \\ -8{,}33\cdot10^{7} \\ 8\cdot10^{4} \\ -1\cdot10^{5} \end{bmatrix}$$

$$\Downarrow$$

$\varphi_B = [illegible]\ 2{,}22\cdot10^{-3}$

$\varphi_C = [illegible]\ -1{,}15\cdot10^{-3}$

$\delta_B = [illegible]\ 3{,}86$ mm

$\delta_D = [illegible]\ 8{,}15$ mm

Momenti

$M_{AB} = -102,17$ KNm

$M_{BA} = 37,02$ KNm

$M_{BC} = -37,00$ KNm

$M_{CB} = -87,37$ KNm

$M_{CD} = 87,37$ KNm

$M_{DC} = 138,08$ KNm

Tagli

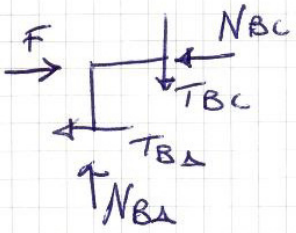
$$T_{AB} = T_{BA} = -\frac{M_{AB}+M_{BC}}{H} = 21,72\,KN$$

$$T_{BC} = \frac{qL}{2} - \frac{M_{BC}+M_{CB}}{L} = 124,87\,KN$$

$$T_{CB} = -\frac{qL}{2} - \frac{M_{BC}-M_{CB}}{L} = -75,13\,KN$$

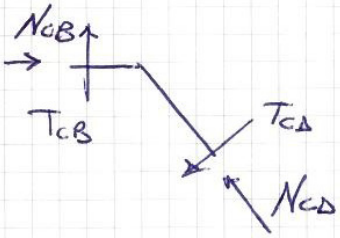
$$T_{CD} = T_{DC} = -\frac{M_{CD}+M_{DC}}{\sqrt{2}H} = -53,14\,KN$$

Sforzi normali



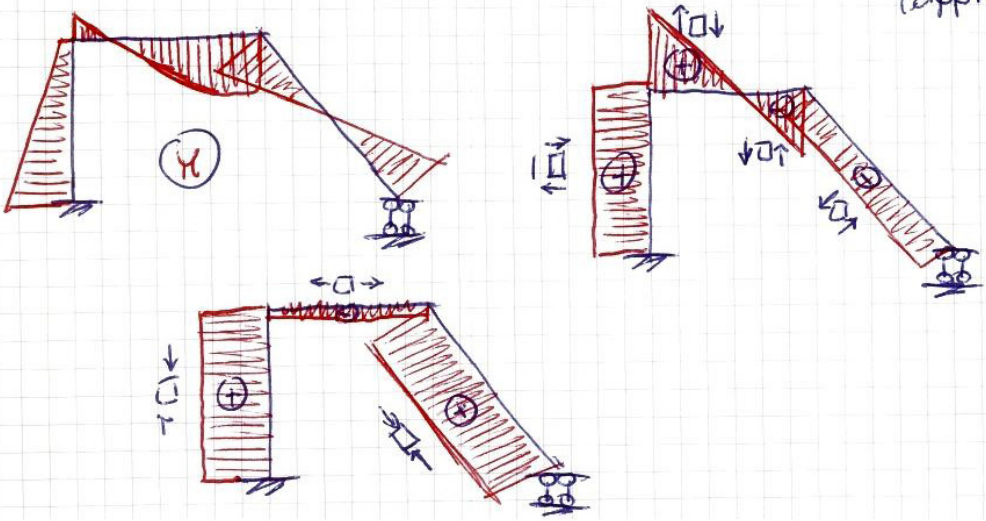
$N_{BA} = T_{BC} = 124,87$ KN

$N_{BC} = F - T_{BA} = 20 - 21,72$ KN $= -1,72$ KN



$$\frac{N_{CD}}{\sqrt{2}} - \frac{T_{CD}}{\sqrt{2}} + T_{CB} = 0$$

$$N_{CD} = \left(\frac{T_{CD}}{\sqrt{2}} - T_{CB}\right)\sqrt{2} = 53,11\,KN$$

$$N_{CB} = \frac{T_{CD}}{\sqrt{2}} + \frac{N_{CD}}{\sqrt{2}} = -0,03KN \quad OK \approx N_{BC}$$

(appr. di calcolo)