

Caratteristiche geometriche

$$L_1 = 4.50 + 0.15 N - 0.10 C = \mathbf{5.15 \text{ m}}$$

$$L_2 = 5.00 + 0.15 N - 0.10 C = \mathbf{5.65 \text{ m}}$$

$$L_3 = 4.00 + 0.10 N - 0.10 C = \mathbf{4.30 \text{ m}}$$

$$L_s = 1.05 + 0.05 M = \mathbf{1.2 \text{ m}}$$

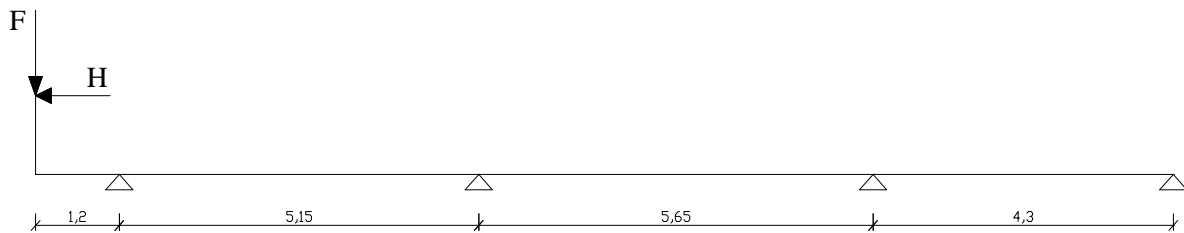
$$B = \mathbf{50 \text{ cm}} \ ; \ b = \mathbf{10 \text{ cm}} \ ; \ s = \mathbf{4 \text{ cm}} \ ; \ h = L_{\max} / 25$$

Caratteristiche meccaniche

Calcestruzzo : $R_{ck} = 25.0 \text{ MPa}$

Acciaio : FeB38K

Schema



Analisi dei carichi

$$h = L_{\max} / 25 = \frac{5.65}{25} = 0.226 \text{ m} = 22.6 \text{ cm}$$

Campata centrale ($h = 24 \text{ cm}$; $s = 4 \text{ cm}$; $h_1 = 20 \text{ cm}$)

- travetti	$(0.20 \times 0.10 \times 1) \times 2 = 0.04 \times 25 =$	1.00 kN/m ²
- soletta	$(0.04 \times 1 \times 1) = 0.04 \times 25 =$	1.00 kN/m ²
- laterizi	$(0.20 \times 0.40 \times 1) \times 2 = 0.16 \times 8 =$	1.28 kN/m ²

peso proprio $g_k = 3.28 \text{ kN/m}^2$

- 3 cm massetto	$(0.03 \times 1 \times 1) = 0.03 \times 18 =$	0.54 kN/m ²
- 2 cm pavimento	$(0.02 \times 1 \times 1) = 0.02 \times 20 =$	0.40 kN/m ²
- 1.5 cm intonaco	$(0.015 \times 1 \times 1) = 0.015 \times 20 =$	0.30 kN/m ²
- incidenza tramezzi		1.00 kN/m ²

sovraccarichi permanenti $g'_k = 2.24 \text{ kN/m}^2$

sovraccarichi accidentali $q_k = 3.00 \text{ kN/m}^2$

Sbalzo ($h_{sb} = 20 \text{ cm}$; $s = 4 \text{ cm}$; $h_1 = 16 \text{ cm}$)

- travetti	$(0.16 \times 0.10 \times 1) \times 2 = 0.032 \times 25 =$	0.80 kN/m ²
- soletta	$(0.04 \times 1 \times 1) = 0.04 \times 25 =$	1.00 kN/m ²
- laterizi	$(0.16 \times 0.4 \times 1) \times 2 = 0.128 \times 8 =$	1.02 kN/m ²

peso proprio $g_{sk} = 2.82 \text{ kN/m}^2$

- 3 cm massetto	$(0.03 \times 1 \times 1) = 0.03 \times 18 =$	0.54 kN/m ²
- 2 cm pavimento	$(0.02 \times 1 \times 1) = 0.02 \times 20 =$	0.40 kN/m ²
- 1.5 cm intonaco	$(0.015 \times 1 \times 1) = 0.015 \times 20 =$	0.30 kN/m ²

sovraccarichi permanenti $g'_{sk} = 1.24 \text{ kN/m}^2$

- ringhiera $F = 0.50 \text{ kN/m}$

sovraccarichi accidentali $q_{sk} = 4 \text{ kN/m}^2$

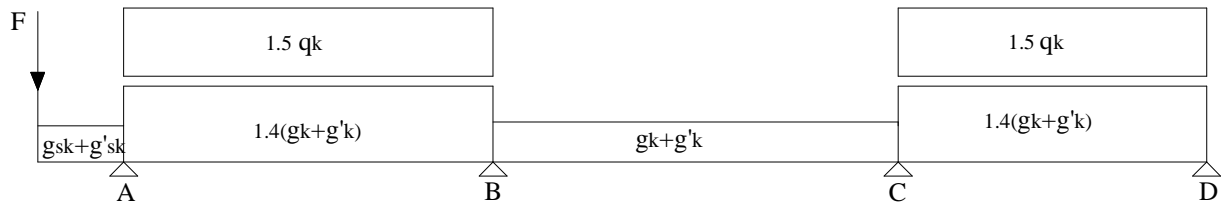
$H = 1.00 \text{ kN/m}$

Fattori di amplificazione dei carichi

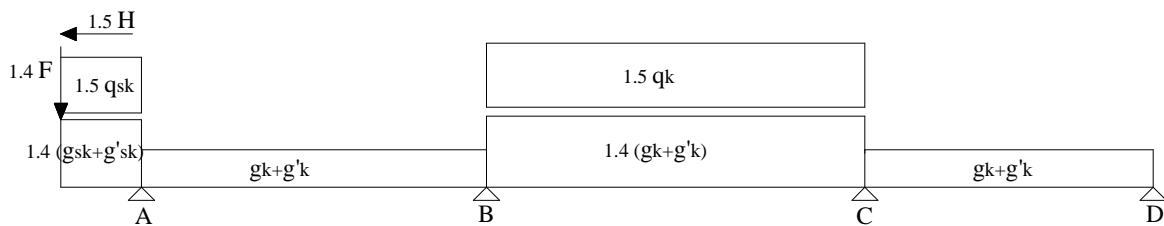
$\gamma_g = 1.4$

$\gamma_q = 1.5$

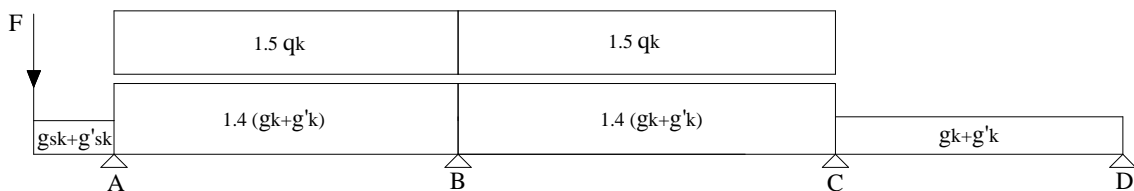
combinazione S.L.U. 1



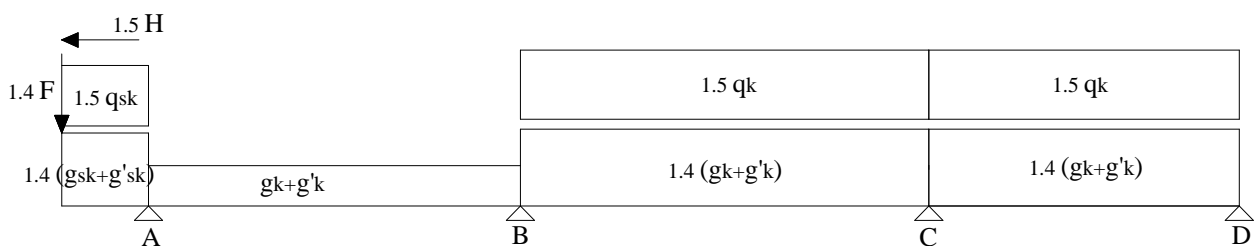
combinazione S.L.U. 2



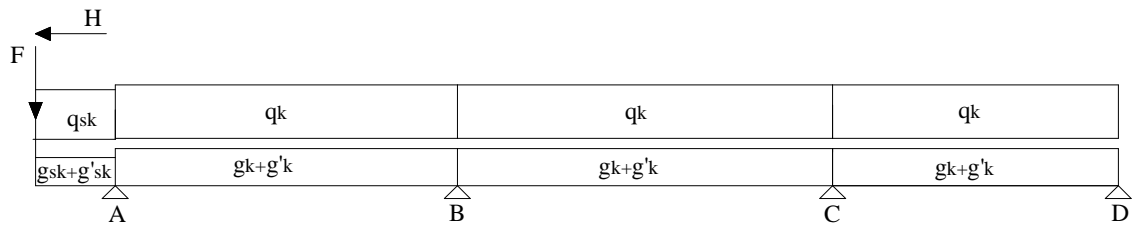
combinazione S.L.U. 3



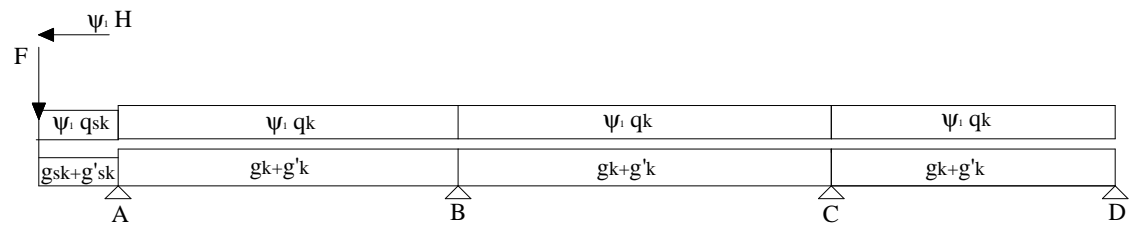
combinazione S.L.U. 4



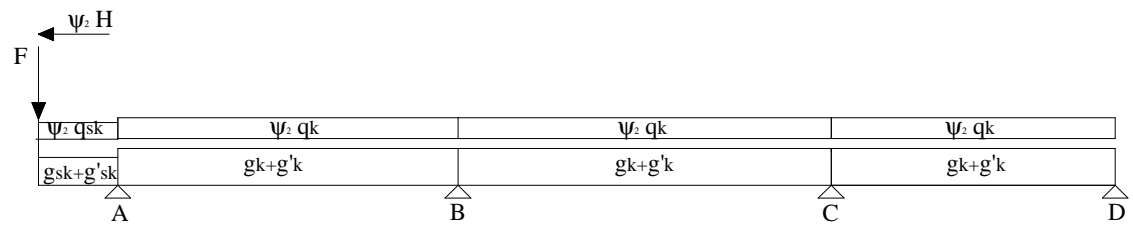
combinazione S.L.S. 1 (rara)



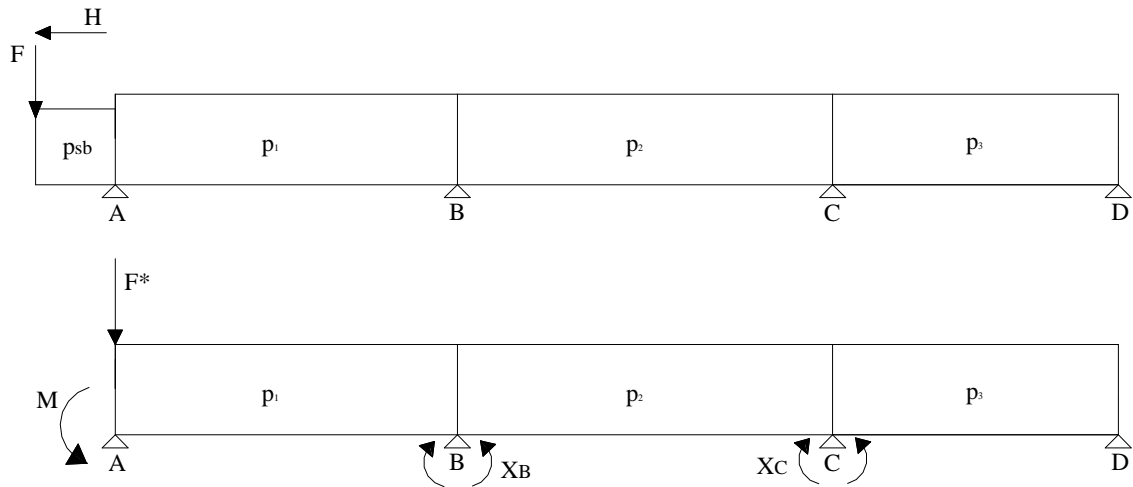
combinazione S.L.S. 2 (frequente) $\psi_1 = 0.6$



combinazione S.L.S. 3 (quasi permanente) $\psi_2 = 0.3$



Risoluzione con il metodo delle forze



$$F^* = F + p_{sb} \cdot L_{sb}$$

$$M = F \cdot L_{sb} + p_{sb} \cdot \frac{L_{sb}^2}{2} + H \cdot h$$

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI}$$

$$\varphi_{CB} = \frac{X_B L_2}{6EI} - \frac{p_2 L_2^3}{24EI} + \frac{X_C L_2}{3EI}$$

$$\varphi_{CD} = \frac{p_3 L_3^3}{24EI} - \frac{X_C L_3}{3EI}$$

$$\varphi_{BA} - \varphi_{BC} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} - \frac{p_2 L_2^3}{24EI} + \frac{X_B L_2}{3EI} + \frac{X_C L_2}{6EI} = 0$$

$$\varphi_{CB} - \varphi_{CD} = \frac{X_B L_2}{6EI} - \frac{p_2 L_2^3}{24EI} + \frac{X_C L_2}{3EI} - \frac{p_3 L_3^3}{24EI} + \frac{X_C L_3}{3EI} = 0$$

$$\frac{X_B L_1}{3EI} + \frac{X_B L_2}{3EI} + \frac{X_C L_2}{6EI} = \frac{p_1 L_1^3}{24EI} + \frac{p_2 L_2^3}{24EI} - \frac{M L_1}{6EI}$$

$$\frac{X_B L_2}{6EI} + \frac{X_C L_2}{3EI} + \frac{X_C L_3}{3EI} = \frac{p_2 L_2^3}{24EI} + \frac{p_3 L_3^3}{24EI}$$

portando in forma matriciale e semplificando EI :

$$\begin{bmatrix} \frac{L_1}{3} + \frac{L_2}{3} & \frac{L_2}{6} \\ \frac{L_2}{6} & \frac{L_2}{3} + \frac{L_3}{3} \end{bmatrix} \times \begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} \frac{p_1 L_1^3}{24} + \frac{p_2 L_2^3}{24} - \frac{M L_1}{6} \\ \frac{p_2 L_2^3}{24} + \frac{p_3 L_3^3}{24} \end{bmatrix}$$

$$\mathbf{D}_X \mathbf{X}^{\text{comb}} = \boldsymbol{\delta}^{\text{comb}} \Rightarrow \mathbf{X}^{\text{comb}} = \mathbf{D}^{-1}_X \boldsymbol{\delta}^{\text{comb}}$$

$$\mathbf{D} = \begin{bmatrix} 3.6000 & 0.9417 \\ 0.9417 & 3.3167 \end{bmatrix}$$

$$\mathbf{D}^{-1} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix}$$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} \times \begin{bmatrix} \frac{p_1 L_1^3}{24} + \frac{p_2 L_2^3}{24} - \frac{M L_1}{6} \\ \frac{p_2 L_2^3}{24} + \frac{p_3 L_3^3}{24} \end{bmatrix}$$

Luci delle varie campate

$$L_{SB} = 1.20 \text{ m}$$

$$L_1 = 5.15 \text{ m}$$

$$L_2 = 5.65 \text{ m}$$

$$L_3 = 4.30 \text{ m}$$

Carichi

$$g_{sk} + g'_{sk} = 4.06 \text{ kN/m}$$

$$g_k + g'_k = 5.52 \text{ kN/m}$$

$$q_{sk} = 4.00 \text{ kN/m}$$

$$q_k = 3.00 \text{ kN/m}$$

$$F = 0.50 \text{ kN}$$

$$H = 1.00 \text{ kN}$$

Combinazione S.L.U. 1

$$p_{sb} = 4.06 \text{ kN/m}$$

$$M = 3.52 \text{ kNm}$$

$$F^* = 5.37 \text{ kN}$$

$$p_1 = 12.23 \text{ kN/m}$$

$$p_2 = 5.52 \text{ kN/m}$$

$$p_3 = 12.23 \text{ kN/m}$$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 108.0664 \\ 81.9987 \end{bmatrix} \Rightarrow \begin{bmatrix} X_B^{(1)} = 25.44kNm \\ X_C^{(1)} = 17.50kNm \end{bmatrix}$$

Combinazione S.L.U. 2

$$\begin{aligned} p_{sb} &= 11.68 \text{ kN/m} & M &= 10.75 \text{ kNm} \\ F^* &= 14.72 \text{ kN} & p_1 &= 5.52 \text{ kN/m} \\ & & p_2 &= 12.23 \text{ kN/m} \\ & & p_3 &= 5.52 \text{ kN/m} \end{aligned}$$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 114.0898 \\ 110.1961 \end{bmatrix} \Rightarrow \begin{bmatrix} X_B^{(2)} = 24.85kNm \\ X_C^{(2)} = 26.17kNm \end{bmatrix}$$

Combinazione S.L.U. 3

$$\begin{aligned} p_{sb} &= 4.06 \text{ kN/m} & M &= 3.52 \text{ kNm} \\ F^* &= 5.37 \text{ kN} & p_1 &= 12.23 \text{ kN/m} \\ & & p_2 &= 12.23 \text{ kN/m} \\ & & p_3 &= 5.52 \text{ kN/m} \end{aligned}$$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 158.4926 \\ 110.1961 \end{bmatrix} \Rightarrow \begin{bmatrix} X_B^{(3)} = 38.17kNm \\ X_C^{(3)} = 22.39kNm \end{bmatrix}$$

Combinazione S.L.U. 4

$$\begin{aligned} p_{sb} &= 11.68 \text{ kN/m} & M &= 10.75 \text{ kNm} \\ F^* &= 14.72 \text{ kN} & p_1 &= 5.52 \text{ kN/m} \\ & & p_2 &= 12.23 \text{ kN/m} \\ & & p_3 &= 12.23 \text{ kN/m} \end{aligned}$$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 114.0984 \\ 133.4250 \end{bmatrix} \Rightarrow \begin{bmatrix} X_B^{(4)} = 22.96kNm \\ X_C^{(4)} = 33.41kNm \end{bmatrix}$$

Combinazione S.L.S. 1 (rara)

$p_{sb} = 8.06 \text{ kN/m}$

$M = 7.40 \text{ kNm}$

$F^* = 10.17 \text{ kN}$

$p_1 = 8.52 \text{ kN/m}$

$p_2 = 8.52 \text{ kN/m}$

$p_3 = 8.52 \text{ kN/m}$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 106.1666 \\ 92.2535 \end{bmatrix} \Rightarrow \begin{bmatrix} X_B^{(R)} = 24.00 \text{ kNm} \\ X_C^{(R)} = 21.00 \text{ kNm} \end{bmatrix}$$

Combinazione S.L.S. 2 (frequente)

$p_{sb} = 6.46 \text{ kN/m}$

$M = 5.85 \text{ kNm}$

$F^* = 8.25 \text{ kN}$

$p_1 = 7.32 \text{ kN/m}$

$p_2 = 7.32 \text{ kN/m}$

$p_3 = 7.32 \text{ kN/m}$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 91.6494 \\ 79.2601 \end{bmatrix} \Rightarrow \begin{bmatrix} X_B^{(F)} = 20.75 \text{ kNm} \\ X_C^{(F)} = 18.01 \text{ kNm} \end{bmatrix}$$

Combinazione S.L.S. 3 (quasi permanente)

$p_{sb} = 5.26 \text{ kN/m}$

$M = 4.69 \text{ kNm}$

$F^* = 6.81 \text{ kN}$

$p_1 = 6.42 \text{ kN/m}$

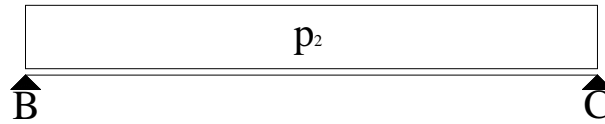
$p_2 = 6.42 \text{ kN/m}$

$p_3 = 6.42 \text{ kN/m}$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 80.7593 \\ 69.5150 \end{bmatrix} \Rightarrow \begin{bmatrix} X_B^{(Q,P)} = 18.31 \text{ kNm} \\ X_C^{(Q,P)} = 15.76 \text{ kNm} \end{bmatrix}$$

Risoluzione con il metodo di Hardy – Cross

Campata intermedia BC



$$\mu_{BC} = -\frac{p_2 L_2^2}{12}$$

$$\mu_{CB} = \frac{p_2 L_2^2}{12}$$

momenti di incastro perfetto

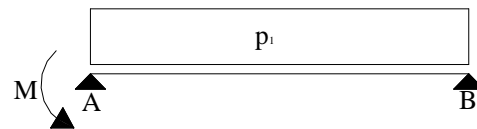
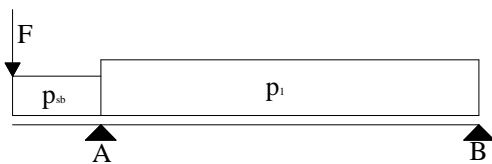
$$\left. \begin{matrix} \varphi_B = 1 \\ \varphi_C = 0 \end{matrix} \right\} \Rightarrow \begin{cases} \frac{W_{BC} L_2}{3EI} - \frac{V_{CB} L_2}{6EI} = 1 \\ -\frac{W_{BC} L_2}{6EI} + \frac{V_{CB} L_2}{3EI} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} W_{BC} = \frac{4EI}{L_2} \\ V_{CB} = \frac{2EI}{L_2} \end{cases} \quad \text{coefficienti di rigidezza}$$

$$\left. \begin{matrix} \varphi_B = 0 \\ \varphi_C = 1 \end{matrix} \right\} \Rightarrow \begin{cases} \frac{V_{BC} L_2}{3EI} - \frac{W_{CB} L_2}{6EI} = 0 \\ -\frac{V_{BC} L_2}{6EI} + \frac{W_{CB} L_2}{3EI} = 1 \end{cases}$$

$$\Rightarrow \begin{cases} V_{BC} = \frac{2EI}{L_2} \\ W_{CB} = \frac{4EI}{L_2} \end{cases} \quad \text{coefficienti di rigidezza}$$

Campata di riva AB



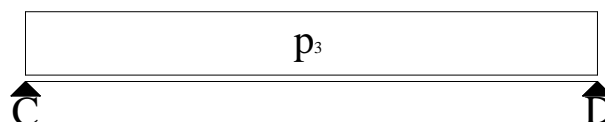
$$\mu_{BA} = \frac{p_1 L_1^2}{8} - \frac{M}{2}$$

momento di incastro perfetto

$$\left[\varphi_B = 1 \right] \Rightarrow \left[\frac{W_{BA} L_1}{3EI} = 1 \right]$$

$$\Rightarrow \left[W_{BA} = \frac{3EI}{L_1} \right] \quad \text{coefficiente di rigidezza}$$

Campata di riva CD



$$\mu_{CD} = -\frac{P_3 L_3^2}{8} \quad \text{momento di incastro perfetto}$$

$$\varphi_C = 1] \Rightarrow \left[\frac{W_{CD} L_3}{3EI} = 1 \right] \Rightarrow \left[W_{CD} = \frac{3EI}{L_3} \right] \quad \text{coefficiente di rigidezza}$$

Essendo $\tau_{ij} = \frac{W_{ij}}{W_{ij} + W_{ik}} \quad \text{coefficiente di ripartizione}$

$$t_{ji} = \frac{V_{ji}}{W_{ij}} \quad \text{coefficiente di trasporto}$$

risulta la tabella

<i>nodo</i>	<i>asta</i>	<i>coeff. di rigidezza (W_{ij})</i>	<i>coeff. di ripartizione (τ_{ij})</i>	<i>coeff. di trasporto (t_{ij})</i>
B	AB	$\frac{3EI}{L_1}$	0.451	0
B	BC	$\frac{4EI}{L_2}$	0.549	0.5
C	BC	$\frac{4EI}{L_2}$	0.504	0.5
C	CD	$\frac{3EI}{L_3}$	0.496	0

Combinazione S.L.U. 1

$$p_{sb} = 4.06 \text{ kN/m}$$

$$M = 3.52 \text{ kNm}$$

$$p_2 = 5.52 \text{ kN/m}$$

$$F^* = 5.37 \text{ kN}$$

$$p_1 = 12.23 \text{ kN/m}$$

$$p_3 = 12.23 \text{ kN/m}$$

$$\mu_{BA} = 38.79 \text{ kNm}$$

$$\mu_{BC} = -14.68 \text{ kNm}$$

$$\mu_{CB} = 14.68 \text{ kNm}$$

$$\mu_{CD} = -28.27 \text{ kNm}$$

	0.451	0.549		0.504	0.496	
A	B			C		D
	38.79	-14.68		14.68	-28.27	
	-10.87	24.11	→	-6.62		
		-13.24	←	-20.21		
		5.09	→	10.19	10.02	
	-2.30	-2.79	→	-1.39		
		0.35	←	0.70	0.69	
	-0.16	-0.19	→	-0.09		
		0.02	←	0.05	0.04	
	-0.01	-0.01				
	25.45	-25.45		17.52	-17.52	

$$X_B^{(1)} = 25.45 \text{ kNm}$$

$$X_C^{(1)} = 17.52 \text{ kNm}$$

Verifica di congruenza

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} = \frac{3.0213}{EI} - \frac{69.6044}{EI} + \frac{43.6892}{EI} = -\frac{22.8939}{EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI} = \frac{41.4833}{EI} - \frac{47.9308}{EI} - \frac{16.4980}{EI} = -\frac{22.9455}{EI}$$

$$\varphi^* = \frac{\varphi_{BA} + \varphi_{BC}}{2} = -\frac{22.9197}{EI}$$

$$\varepsilon = \frac{|\varphi_{BC} - \varphi_{BA}|}{|\varphi^*|} = 2.25 \times 10^{-3}$$

Combinazione S.L.U. 2

$$p_{sb} = 11.68 \text{ kN/m}$$

$$M = 10.75 \text{ kNm}$$

$$p_2 = 12.23 \text{ kN/m}$$

$$F^* = 14.72 \text{ kN}$$

$$p_1 = 5.52 \text{ kN/m}$$

$$p_3 = 5.52 \text{ kN/m}$$

$$\mu_{BA} = 12.93 \text{ kNm}$$

$$\mu_{BC} = -32.53 \text{ kNm}$$

$$\mu_{CD} = -12.76 \text{ kNm}$$

$$\mu_{CB} = 32.53 \text{ kNm}$$

	0.451	0.549		0.504	0.496
	B			C	
	12.93	-32.53		32.53	-12.76
		-19.60			
	8.84	10.76	→	5.38	
		-6.34	←	25.15	
	2.86	3.48	→	-12.68	-12.47
		-0.44	←	1.74	
	0.20	0.24	→	-0.88	-0.86
		-0.03	←	0.12	
	0.01	0.02	→	-0.06	-0.06
				0.01	
				-0.01	-0.00
	24.84	-24.84		26.15	-26.15

$$X_B^{(2)} = 24.84 \text{ kNm}$$

$$X_C^{(2)} = 26.15 \text{ kNm}$$

Verifica di congruenza

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} = \frac{9.2271}{EI} - \frac{31.4159}{EI} + \frac{42.6420}{EI} = + \frac{20.4532}{EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI} = \frac{91.9095}{EI} - \frac{46.482}{EI} - \frac{24.6246}{EI} = + \frac{20.8029}{EI}$$

$$\varphi^* = \frac{\varphi_{BA} + \varphi_{BC}}{2} = + \frac{20.6281}{EI}$$

$$\varepsilon = \frac{|\varphi_{BC} - \varphi_{BA}|}{|\varphi^*|} = 1.69 \times 10^{-2}$$

Combinazione S.L.U. 3

$$p_{sb} = 4.06 \text{ kN/m}$$

$$M = 3.52 \text{ kNm}$$

$$p_2 = 12.23 \text{ kN/m}$$

$$F^* = 5.37 \text{ kN}$$

$$p_1 = 12.23 \text{ kN/m}$$

$$p_3 = 5.52 \text{ kN/m}$$

$$\mu_{BA} = 38.79 \text{ kNm}$$

$$\mu_{BC} = -32.53 \text{ kNm}$$

$$\mu_{CD} = -12.76 \text{ kNm}$$

$$\mu_{CB} = 32.53 \text{ kNm}$$

		0.451	0.549		0.504	0.496	
▲	B			C			▲
A		38.79	-32.53		32.53	-12.76	D
		-2.82	6.26 -3.44	→	-1.72		
			-4.55	←	18.05		
		2.05	2.50	→	-9.10	-8.95	
			-0.31	←	1.25		
		0.14	0.17	→	-0.63	-0.62	
			-0.02	←	0.08		
		0.01	0.01	→	-0.04	-0.04	
				←			
		38.17	-38.17		22.37	-22.37	

$$X_B^{(3)} = 38.17 \text{ kNm}$$

$$X_C^{(3)} = 22.37 \text{ kNm}$$

Verifica di congruenza

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} = \frac{3.0213}{EI} - \frac{69.6044}{EI} + \frac{65.5252}{EI} = -\frac{1.0579}{EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI} = \frac{91.9095}{EI} - \frac{71.8868}{EI} - \frac{21.0651}{EI} = -\frac{1.0424}{EI}$$

$$\varphi^* = \frac{\varphi_{BA} + \varphi_{BC}}{2} = -\frac{1.0502}{EI}$$

$$\varepsilon = \frac{|\varphi_{BC} - \varphi_{BA}|}{|\varphi^*|} = 1.47 \times 10^{-2}$$

Combinazione S.L.U. 4

$$\begin{aligned}
 p_{sb} &= 11.68 \text{ kN/m} & M &= 10.75 \text{ kNm} & p_2 &= 12.23 \text{ kN/m} \\
 F^* &= 14.72 \text{ kN} & p_1 &= 5.52 \text{ kN/m} & p_3 &= 12.23 \text{ kN/m} \\
 \mu_{BA} &= 12.93 \text{ kNm} \\
 \mu_{BC} &= -32.53 \text{ kNm} & \mu_{CB} &= 32.53 \text{ kNm} \\
 \mu_{CD} &= -28.27 \text{ kNm}
 \end{aligned}$$

	0.451	0.549		0.504	0.496
	B			C	
	12.93	-32.53		32.53	-28.27
		-19.60			
	8.84	10.76	→	5.38	
		-2.43	←	9.64	
	1.10	1.33	→	-4.86	-4.78
		-0.16	←	0.66	
	0.07	0.09	→	-0.33	-0.33
		-0.01	←	0.04	
	0.00	0.01		-0.02	-0.02
	22.94	-22.94		33.40	-33.40

$$X_B^{(4)} = 22.94 \text{ kNm}$$

$$X_C^{(4)} = 33.40 \text{ kNm}$$

Verifica di congruenza

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} = \frac{9.2271}{EI} - \frac{31.4159}{EI} + \frac{39.3803}{EI} = + \frac{17.1915}{EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI} = \frac{91.9095}{EI} - \frac{43.2037}{EI} - \frac{31.4517}{EI} = + \frac{17.2541}{EI}$$

$$\varphi^* = \frac{\varphi_{BA} + \varphi_{BC}}{2} = + \frac{17.2228}{EI}$$

$$\varepsilon = \frac{|\varphi_{BC} - \varphi_{BA}|}{|\varphi^*|} = 3.63 \times 10^{-3}$$

Combinazione S.L.S. 1 (rara)

$$p_{sb} = 8.06 \text{ kN/m}$$

$$M = 7.40 \text{ kNm}$$

$$p_2 = 8.52 \text{ kN/m}$$

$$F^* = 10.17 \text{ kN}$$

$$p_1 = 8.52 \text{ kN/m}$$

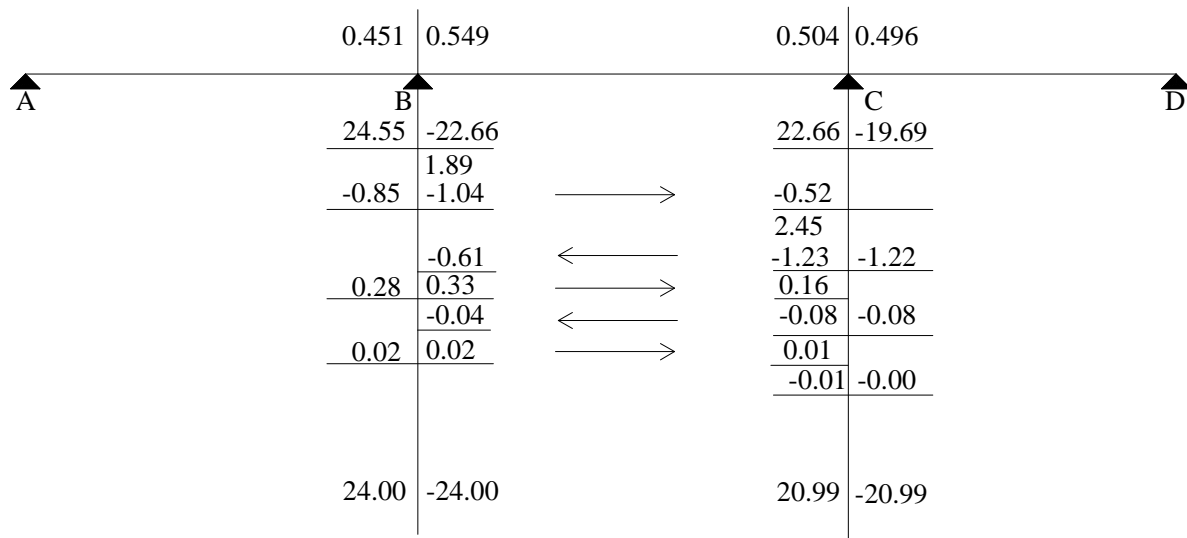
$$p_3 = 8.52 \text{ kN/m}$$

$$\mu_{BA} = 24.55 \text{ kNm}$$

$$\mu_{BC} = -22.66 \text{ kNm}$$

$$\mu_{CD} = -19.69 \text{ kNm}$$

$$\mu_{CB} = 22.66 \text{ kNm}$$



$$X_B^{(R)} = 24.00 \text{ kNm}$$

$$X_C^{(R)} = 20.99 \text{ kNm}$$

Verifica di congruenza

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} = \frac{6.3517}{EI} - \frac{48.4898}{EI} + \frac{41.2000}{EI} = -\frac{0.9381}{EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI} = \frac{64.0285}{EI} - \frac{45.2000}{EI} - \frac{19.7656}{EI} = -\frac{0.9371}{EI}$$

$$\varphi^* = \frac{\varphi_{BA} + \varphi_{BC}}{2} = -\frac{0.9376}{EI}$$

$$\varepsilon = \frac{|\varphi_{BC} - \varphi_{BA}|}{|\varphi^*|} = 1.06 \times 10^{-3}$$

Combinazione S.L.S. 2 (frequente)

$$p_{sb} = 6.46 \text{ kN/m}$$

$$M = 5.85 \text{ kNm}$$

$$p_2 = 7.32 \text{ kN/m}$$

$$F^* = 8.25 \text{ kN}$$

$$p_1 = 7.32 \text{ kN/m}$$

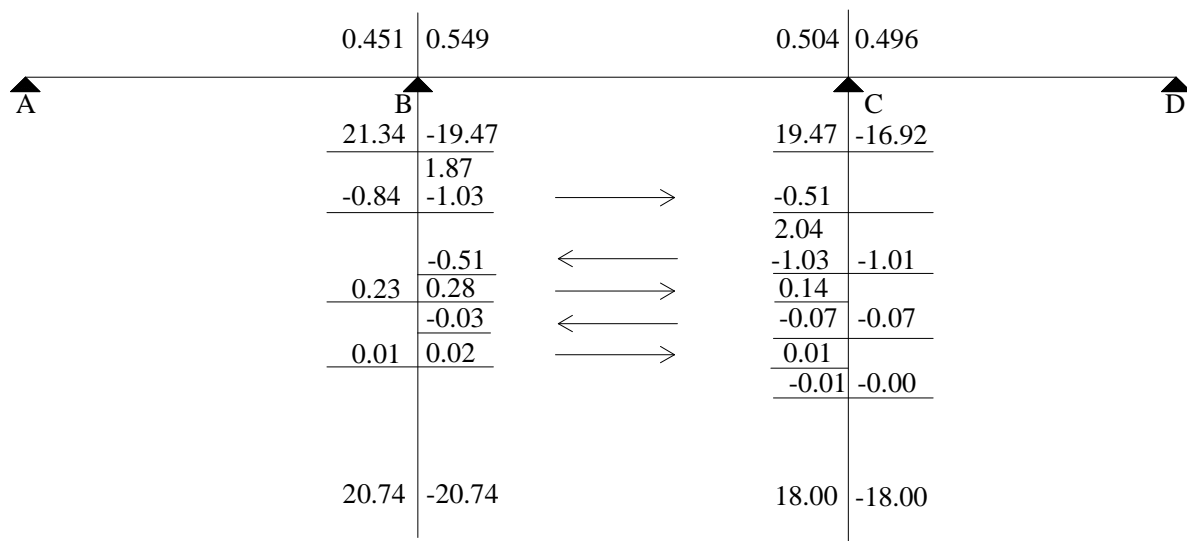
$$p_3 = 7.32 \text{ kN/m}$$

$$\mu_{BA} = 21.34 \text{ kNm}$$

$$\mu_{BC} = -19.47 \text{ kNm}$$

$$\mu_{CD} = -16.92 \text{ kNm}$$

$$\mu_{CB} = 19.47 \text{ kNm}$$



$$X_B^{(F)} = 20.74 \text{ kNm}$$

$$X_C^{(F)} = 18.00 \text{ kNm}$$

Verifica di congruenza

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} = \frac{5.0212}{EI} - \frac{41.6602}{EI} + \frac{35.6037}{EI} = -\frac{1.0353}{EI}$$

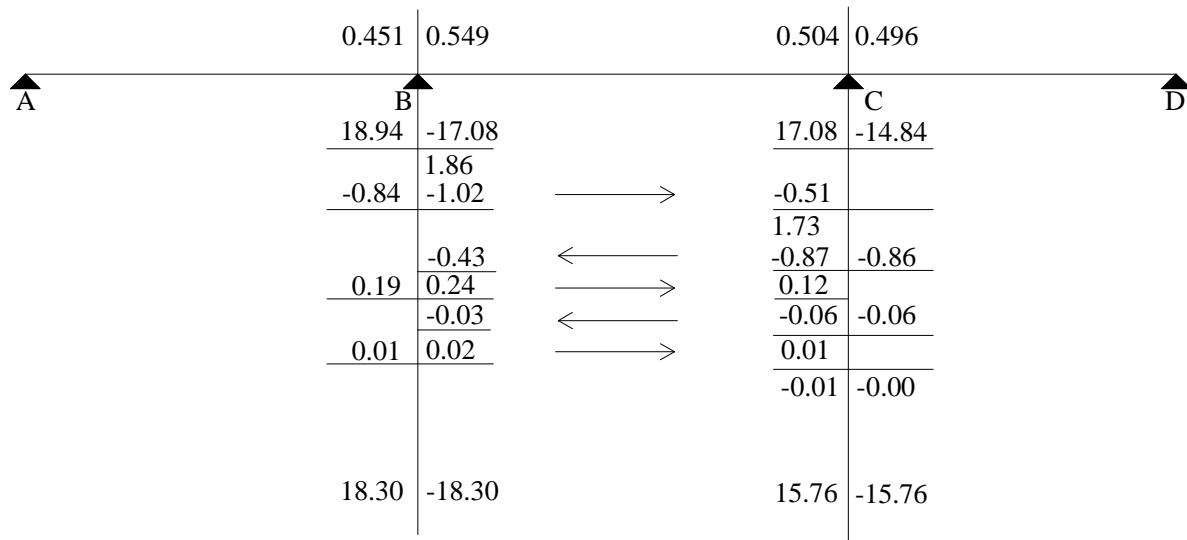
$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI} = \frac{55.0104}{EI} - \frac{39.0603}{EI} - \frac{16.9500}{EI} = -\frac{0.9999}{EI}$$

$$\varphi^* = \frac{\varphi_{BA} + \varphi_{BC}}{2} = -\frac{1.0176}{EI}$$

$$\varepsilon = \frac{|\varphi_{BC} - \varphi_{BA}|}{|\varphi^*|} = 3.48 \times 10^{-2}$$

Combinazione S.L.S. 3 (quasi permanente)

$$\begin{aligned}
 p_{sb} &= 5.26 \text{ kN/m} & M &= 4.69 \text{ kNm} & p_2 &= 6.42 \text{ kN/m} \\
 F^* &= 6.81 \text{ kN} & p_1 &= 6.42 \text{ kN/m} & p_3 &= 6.42 \text{ kN/m} \\
 \mu_{BA} &= 18.94 \text{ kNm} \\
 \mu_{BC} &= -17.08 \text{ kNm} & \mu_{CB} &= 17.08 \text{ kNm} \\
 \mu_{CD} &= -14.84 \text{ kNm}
 \end{aligned}$$



$$X_B^{(Q,P)} = 18.30 \text{ kNm}$$

$$X_C^{(Q,P)} = 15.76 \text{ kNm}$$

Verifica di congruenza

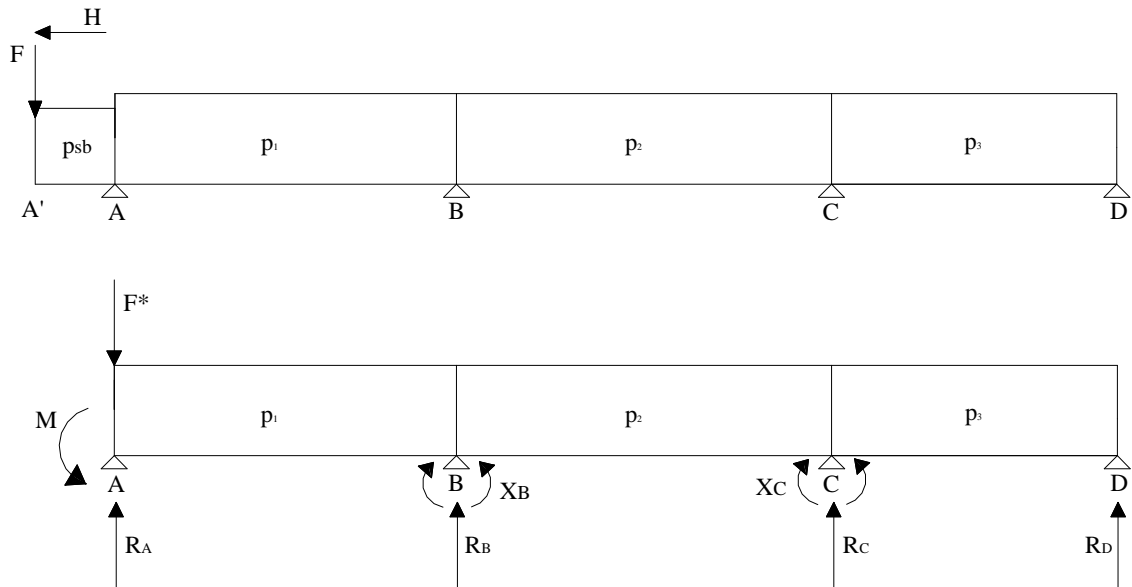
$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI} = \frac{4.0256}{EI} - \frac{36.5380}{EI} + \frac{31.4150}{EI} = -\frac{1.0974}{EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI} = \frac{48.2469}{EI} - \frac{34.4650}{EI} - \frac{14.8407}{EI} = -\frac{1.0588}{EI}$$

$$\varphi^* = \frac{\varphi_{BA} + \varphi_{BC}}{2} = -\frac{1.0781}{EI}$$

$$\varepsilon = \frac{|\varphi_{BC} - \varphi_{BA}|}{|\varphi^*|} = 3.58 \times 10^{-2}$$

Diagrammi di Taglio e Momento



Reazioni vincolari

$$R_A = \frac{M}{L_1} + \frac{p_1 L_1}{2} - \frac{X_B}{L_1} + F^*$$

$$R_B = -\frac{M}{L_1} + \frac{p_1 L_1}{2} + \frac{X_B}{L_1} + \frac{X_B}{L_2} + \frac{p_2 L_2}{2} - \frac{X_C}{L_2}$$

$$R_C = \frac{X_C}{L_2} + \frac{p_2 L_2}{2} - \frac{X_B}{L_2} + \frac{X_C}{L_3} + \frac{p_3 L_3}{2}$$

$$R_D = \frac{p_3 L_3}{2} - \frac{X_C}{L_3}$$

Sollecitazioni di taglio

$$T_A^{(S)} = -F^*$$

$$T_A^{(D)} = T_A^{(S)} + R_A$$

$$T_B^{(S)} = T_A^{(D)} - p_1 L_1$$

$$T_B^{(D)} = T_B^{(S)} + R_B$$

$$T_C^{(S)} = T_B^{(D)} - p_2 L_2$$

$$T_C^{(D)} = T_C^{(S)} + R_C$$

$$T_D^{(S)} = T_C^{(D)} - p_3 L_3 = -R_D$$

Sollecitazioni di momento

$$M_{A'} = -H \times 1.00 \text{ m}$$

$$M_A = -M$$

$$M_B = -X_B$$

$$M_C = -X_C$$

$$M_D = 0$$

Combinazione S.L.U. 1

$$p_{sb} = 4.06 \text{ kN/m}$$

$$M = 3.52 \text{ kNm}$$

$$L_{SB} = 1.20 \text{ m}$$

$$F^* = 5.37 \text{ kN}$$

$$p_1 = 12.23 \text{ kN/m}$$

$$L_1 = 5.15 \text{ m}$$

$$X_B^{(1)} = 25.44 \text{ kNm}$$

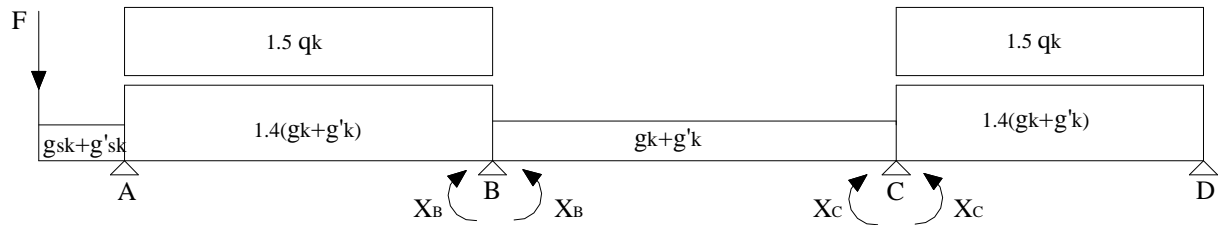
$$p_2 = 5.52 \text{ kN/m}$$

$$L_2 = 5.65 \text{ m}$$

$$X_C^{(1)} = 17.50 \text{ kNm}$$

$$p_3 = 12.23 \text{ kN/m}$$

$$L_3 = 4.30 \text{ m}$$



$$R_A = 32.61 \text{ kN}$$

$$T_A^{(S)} = -5.37 \text{ kN}$$

$$M_A' = 0$$

$$R_B = 52.75 \text{ kN}$$

$$T_A^{(D)} = 27.24 \text{ kN}$$

$$M_A = -3.52 \text{ kNm}$$

$$R_C = 44.55 \text{ kN}$$

$$T_B^{(S)} = -35.75 \text{ kN}$$

$$M_B = -25.44 \text{ kNm}$$

$$R_D = 22.22 \text{ kN}$$

$$T_B^{(D)} = 17.00 \text{ kN}$$

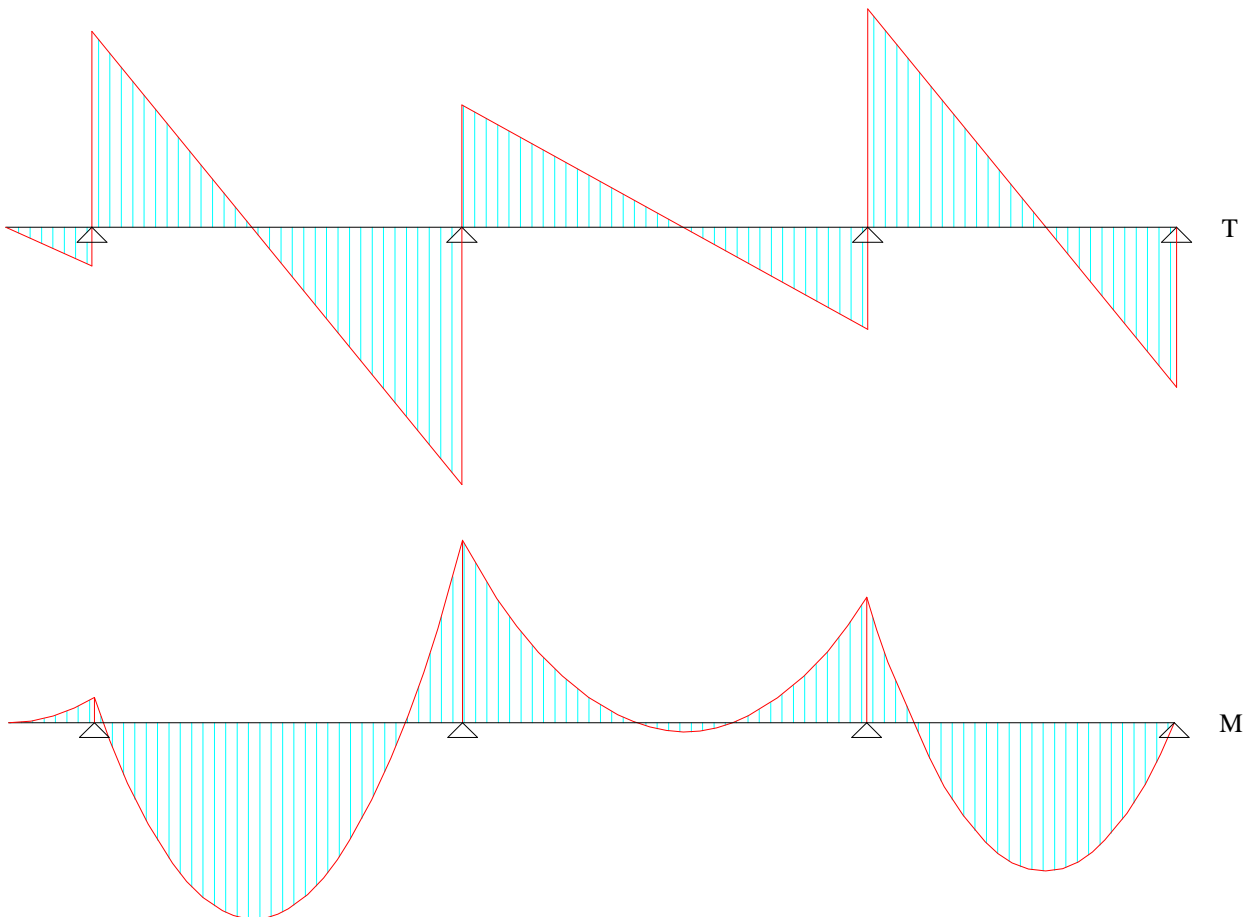
$$M_C = -17.50 \text{ kNm}$$

$$T_C^{(S)} = -14.19 \text{ kN}$$

$$M_D = 0$$

$$T_C^{(D)} = 30.36 \text{ kN}$$

$$T_D^{(S)} = -22.22 \text{ kN}$$



Combinazione S.L.U. 2

$$p_{sb} = 11.68 \text{ kN/m}$$

$$M = 10.75 \text{ kNm}$$

$$H^* = 1.5 \text{ kN}$$

$$L_{SB} = 1.20 \text{ m}$$

$$F^* = 14.72 \text{ kN}$$

$$p_1 = 5.52 \text{ kN/m}$$

$$L_1 = 5.15 \text{ m}$$

$$X_B^{(2)} = 24.85 \text{ kNm}$$

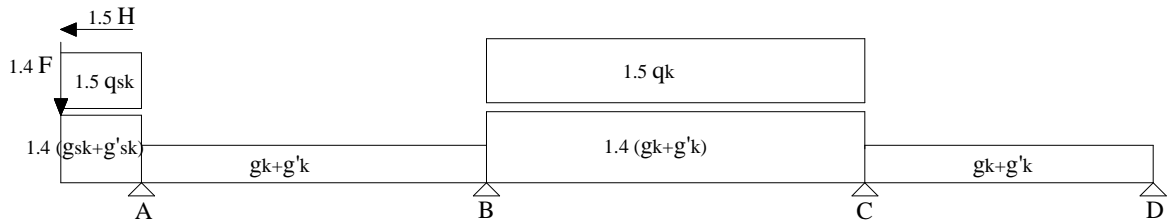
$$p_2 = 12.23 \text{ kN/m}$$

$$L_2 = 5.65 \text{ m}$$

$$X_C^{(2)} = 26.17 \text{ kNm}$$

$$p_3 = 5.52 \text{ kN/m}$$

$$L_3 = 4.30 \text{ m}$$



$$R_A = 26.20 \text{ kN}$$

$$T_A^{(S)} = -14.72 \text{ kN}$$

$$M_A' = -1.5 \text{ kNm}$$

$$R_B = 51.27 \text{ kN}$$

$$T_A^{(D)} = 11.48 \text{ kN}$$

$$M_A = -10.75 \text{ kNm}$$

$$R_C = 52.74 \text{ kN}$$

$$T_B^{(S)} = -16.95 \text{ kN}$$

$$M_B = -24.85 \text{ kNm}$$

$$R_D = 5.78 \text{ kN}$$

$$T_B^{(D)} = 34.32 \text{ kN}$$

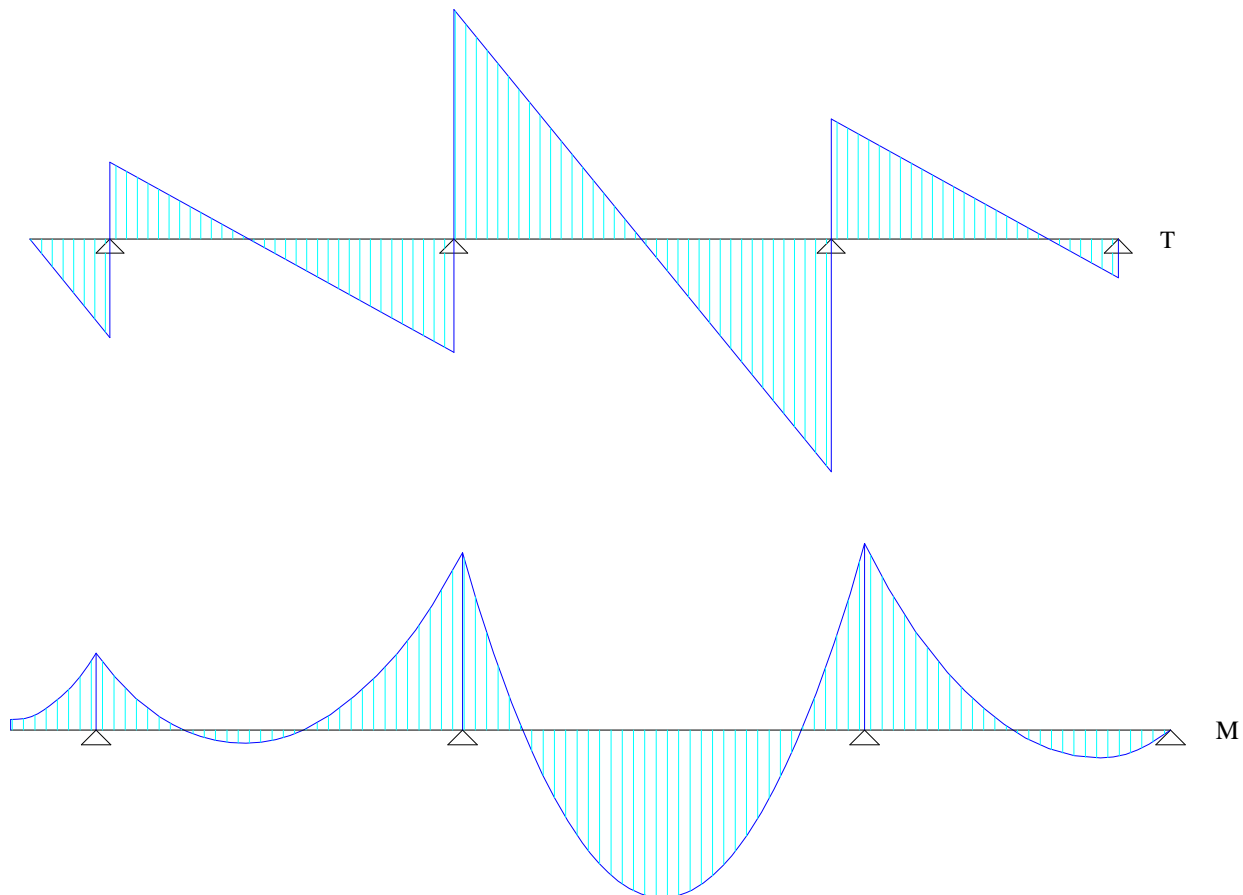
$$M_C = -26.17 \text{ kNm}$$

$$T_C^{(S)} = -34.78 \text{ kN}$$

$$M_D = 0$$

$$T_C^{(D)} = 17.96 \text{ kN}$$

$$T_D^{(S)} = -5.78 \text{ kN}$$



Combinazione S.L.U. 3

$$p_{sb} = 4.06 \text{ kN/m}$$

$$F^* = 5.37 \text{ kN}$$

$$X_B^{(3)} = 38.17 \text{ kNm}$$

$$X_C^{(3)} = 22.39 \text{ kNm}$$

$$M = 3.52 \text{ kNm}$$

$$p_1 = 12.23 \text{ kN/m}$$

$$p_2 = 12.23 \text{ kN/m}$$

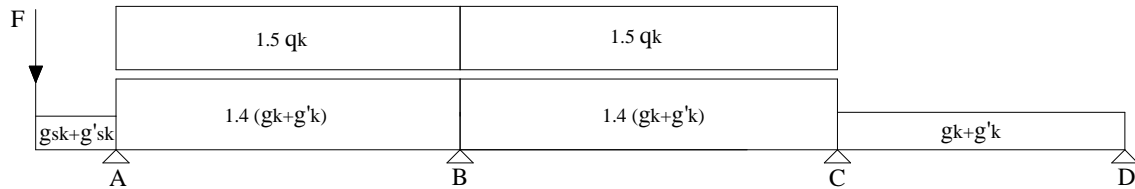
$$p_3 = 5.52 \text{ kN/m}$$

$$L_{SB} = 1.20 \text{ m}$$

$$L_1 = 5.15 \text{ m}$$

$$L_2 = 5.65 \text{ m}$$

$$L_3 = 4.30 \text{ m}$$



$$R_A = 30.13 \text{ kN}$$

$$R_B = 75.56 \text{ kN}$$

$$R_C = 48.83 \text{ kN}$$

$$R_D = 6.66 \text{ kN}$$

$$T_A^{(s)} = -5.37 \text{ kN}$$

$$T_A^{(D)} = 24.76 \text{ kN}$$

$$T_B^{(s)} = -38.22 \text{ kN}$$

$$T_B^{(D)} = 37.34 \text{ kN}$$

$$T_C^{(s)} = -31.76 \text{ kN}$$

$$T_C^{(D)} = 17.07 \text{ kN}$$

$$T_D^{(s)} = -6.67 \text{ kN}$$

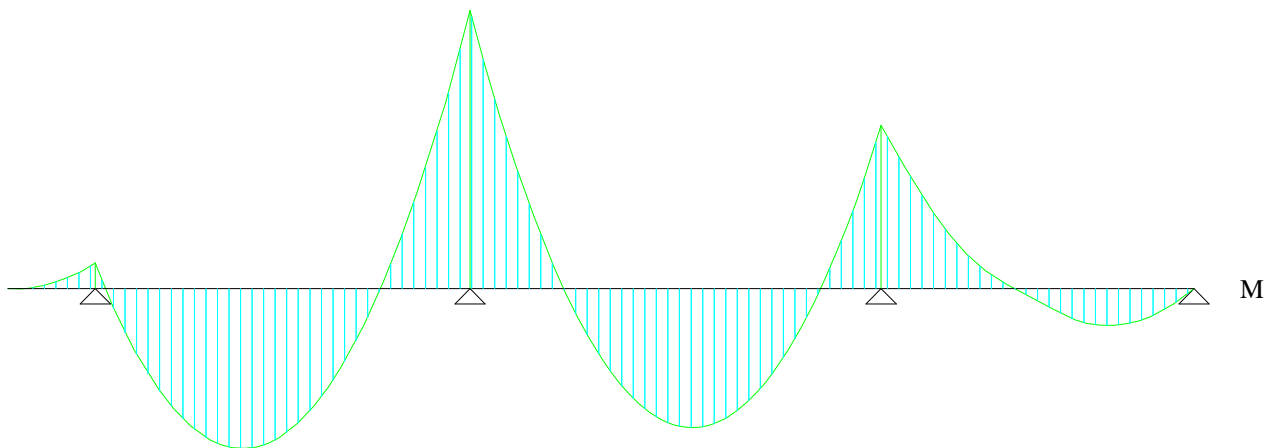
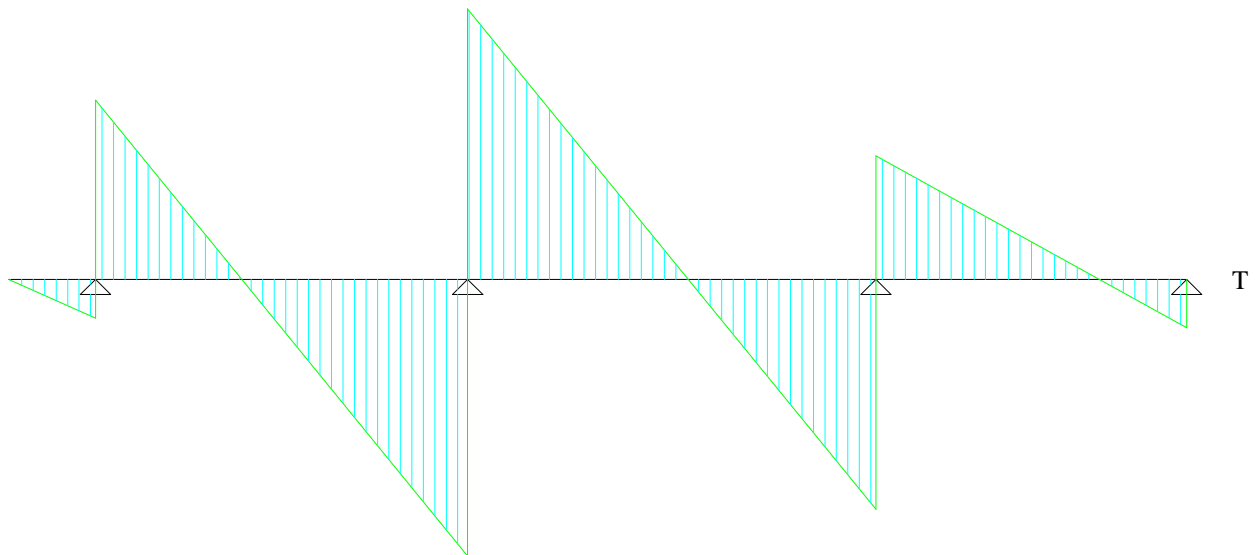
$$M_A' = 0$$

$$M_A = -3.52 \text{ kNm}$$

$$M_B = -38.17 \text{ kNm}$$

$$M_C = -22.39 \text{ kNm}$$

$$M_D = 0$$



Combinazione S.L.U. 4

$$p_{sb} = 11.68 \text{ kN/m}$$

$$M = 10.75 \text{ kNm}$$

$$H^* = 1.5 \text{ kN}$$

$$L_{SB} = 1.20 \text{ m}$$

$$F^* = 14.72 \text{ kN}$$

$$p_1 = 5.52 \text{ kN/m}$$

$$L_1 = 5.15 \text{ m}$$

$$X_B^{(4)} = 22.96 \text{ kNm}$$

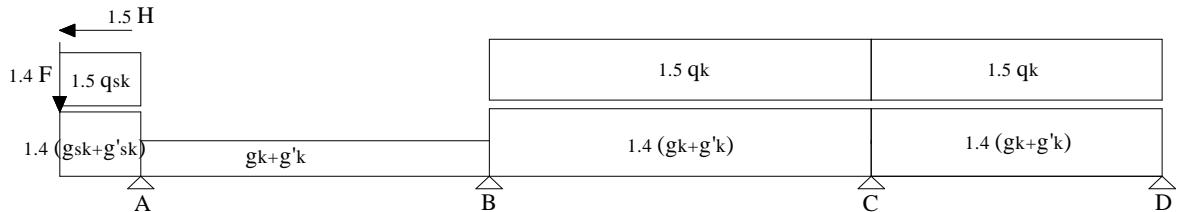
$$p_2 = 12.23 \text{ kN/m}$$

$$L_2 = 5.65 \text{ m}$$

$$X_C^{(4)} = 33.41 \text{ kNm}$$

$$p_3 = 12.23 \text{ kN/m}$$

$$L_3 = 4.30 \text{ m}$$



$$R_A = 26.56 \text{ kN}$$

$$T_A^{(S)} = -14.72 \text{ kN}$$

$$M_A' = -1.5 \text{ kNm}$$

$$R_B = 49.28 \text{ kN}$$

$$T_A^{(D)} = 11.84 \text{ kN}$$

$$M_A = -10.75 \text{ kNm}$$

$$R_C = 70.46 \text{ kN}$$

$$T_B^{(S)} = -16.59 \text{ kN}$$

$$M_B = -22.96 \text{ kNm}$$

$$R_D = 18.52 \text{ kN}$$

$$T_B^{(D)} = 32.69 \text{ kN}$$

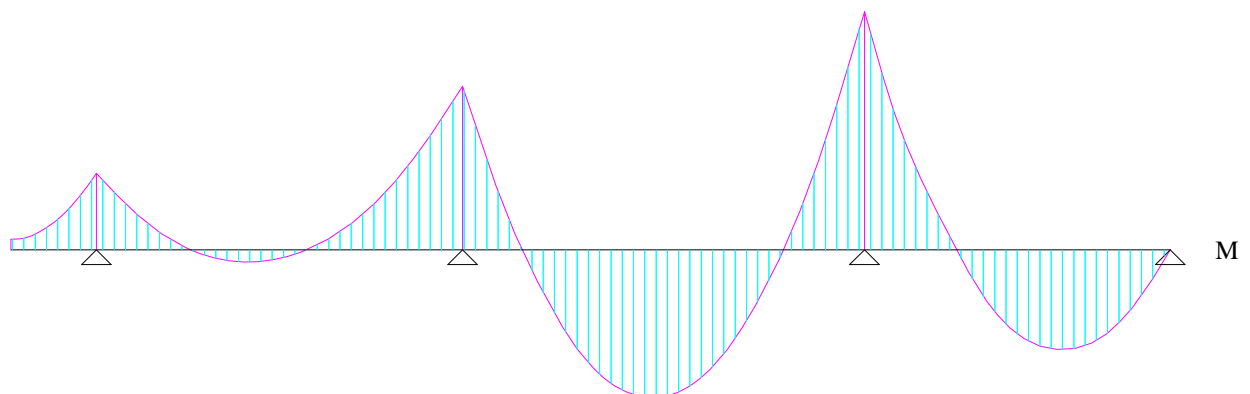
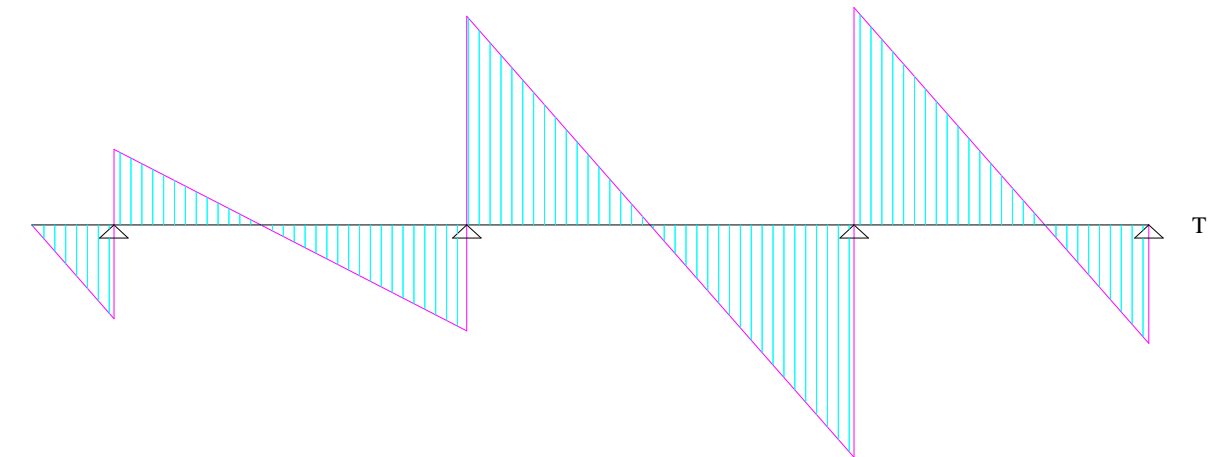
$$M_C = -33.41 \text{ kNm}$$

$$T_C^{(S)} = -36.41 \text{ kN}$$

$$M_D = 0$$

$$T_C^{(D)} = 34.05 \text{ kN}$$

$$T_D^{(S)} = -18.54 \text{ kN}$$



Combinazione S.L.S. 1 (rara)

$$p_{sb} = 8.06 \text{ kN/m}$$

$$M = 7.40 \text{ kNm}$$

$$H = 1.00 \text{ kN}$$

$$L_{SB} = 1.20 \text{ m}$$

$$F^* = 10.17 \text{ kN}$$

$$p_1 = 8.52 \text{ kN/m}$$

$$L_1 = 5.15 \text{ m}$$

$$X_B^{(R)} = 24.00 \text{ kNm}$$

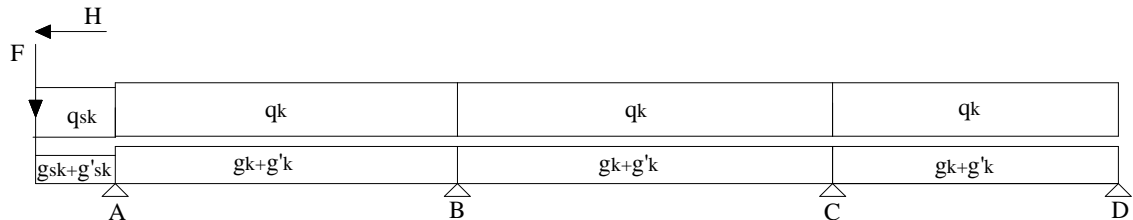
$$p_2 = 8.52 \text{ kN/m}$$

$$L_2 = 5.65 \text{ m}$$

$$X_C^{(R)} = 21.00 \text{ kNm}$$

$$p_3 = 8.52 \text{ kN/m}$$

$$L_3 = 4.30 \text{ m}$$



$$R_A = 28.88 \text{ kN}$$

$$T_A^{(S)} = -10.17 \text{ kN}$$

$$M_A' = -1.00 \text{ kNm}$$

$$R_B = 49.76 \text{ kN}$$

$$T_A^{(D)} = 18.71 \text{ kN}$$

$$M_A = -7.40 \text{ kNm}$$

$$R_C = 46.74 \text{ kN}$$

$$T_B^{(S)} = -25.17 \text{ kN}$$

$$M_B = -24.00 \text{ kNm}$$

$$R_D = 13.43 \text{ kN}$$

$$T_B^{(D)} = 24.59 \text{ kN}$$

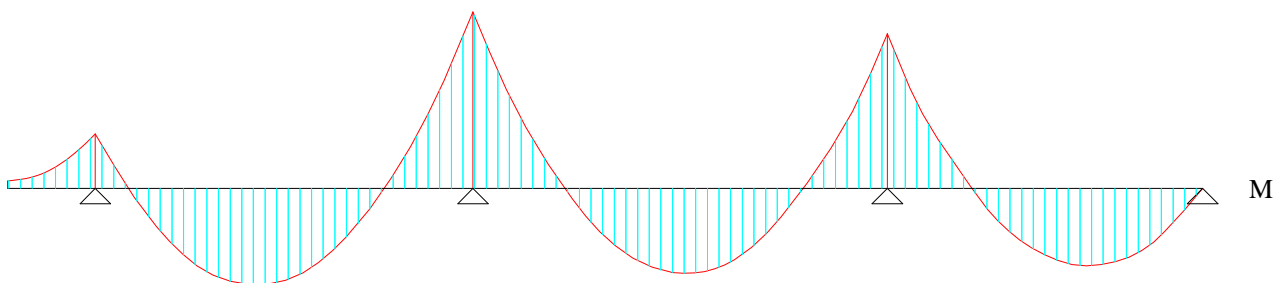
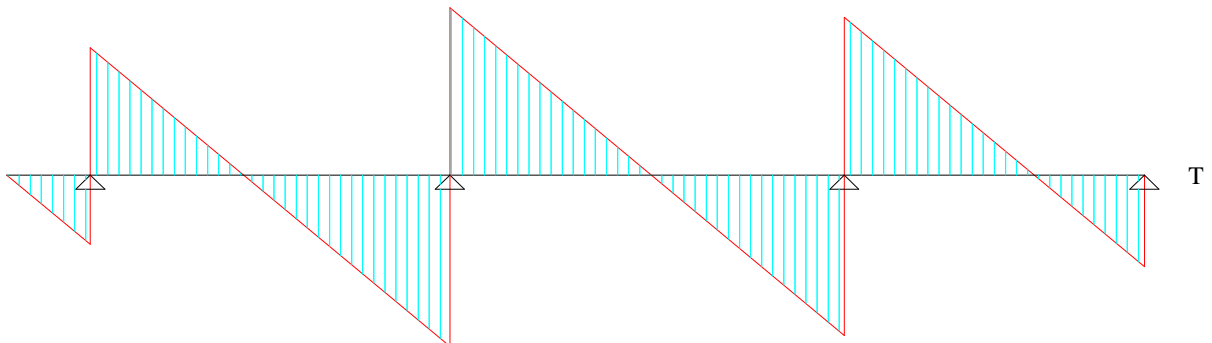
$$M_C = -21.00 \text{ kNm}$$

$$T_C^{(S)} = -23.55 \text{ kN}$$

$$M_D = 0$$

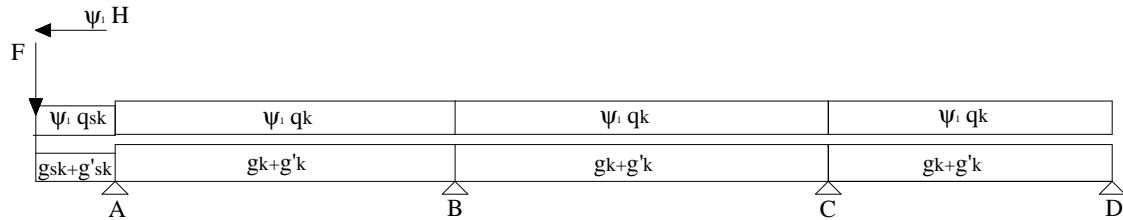
$$T_C^{(D)} = 23.19 \text{ kN}$$

$$T_D^{(S)} = -13.44 \text{ kN}$$

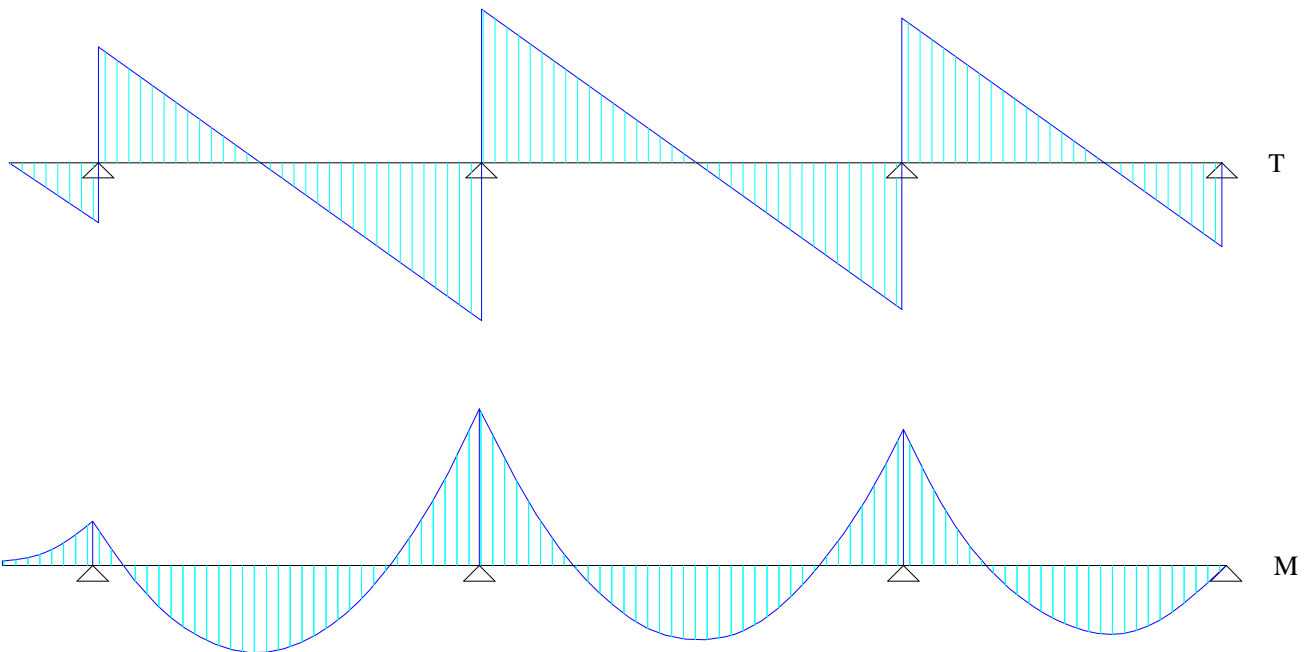


Combinazione S.L.S. 2 (frequente)

$p_{sb} = 6.46 \text{ kN/m}$	$M = 5.85 \text{ kNm}$	$H^* = 0.60 \text{ kN}$	$L_{SB} = 1.20 \text{ m}$
$F^* = 8.25 \text{ kN}$	$p_1 = 7.32 \text{ kN/m}$		$L_1 = 5.15 \text{ m}$
$X_B^{(F)} = 20.75 \text{ kNm}$	$p_2 = 7.32 \text{ kN/m}$		$L_2 = 5.65 \text{ m}$
$X_C^{(F)} = 18.01 \text{ kNm}$	$p_3 = 7.32 \text{ kN/m}$		$L_3 = 4.30 \text{ m}$



$R_A = 24.21 \text{ kN}$	$T_A^{(S)} = -8.25 \text{ kN}$	$M_A' = -0.60 \text{ kNm}$
$R_B = 42.91 \text{ kN}$	$T_A^{(D)} = 15.96 \text{ kN}$	$M_A = -5.85 \text{ kNm}$
$R_C = 40.12 \text{ kN}$	$T_B^{(S)} = -21.74 \text{ kN}$	$M_B = -20.75 \text{ kNm}$
$R_D = 11.55 \text{ kN}$	$T_B^{(D)} = 21.17 \text{ kN}$	$M_C = -18.01 \text{ kNm}$
	$T_C^{(S)} = -20.19 \text{ kN}$	$M_D = 0$
	$T_C^{(D)} = 19.93 \text{ kN}$	
	$T_D^{(S)} = -11.55 \text{ kN}$	



Combinazione S.L.S. 3 (quasi permanente)

$$p_{sb} = 5.26 \text{ kN/m}$$

$$M = 4.69 \text{ kNm}$$

$$H^* = 0.30 \text{ kN}$$

$$L_{SB} = 1.20 \text{ m}$$

$$F^* = 6.81 \text{ kN}$$

$$p_1 = 6.42 \text{ kN/m}$$

$$L_1 = 5.15 \text{ m}$$

$$X_B^{(Q,P)} = 18.31 \text{ kNm}$$

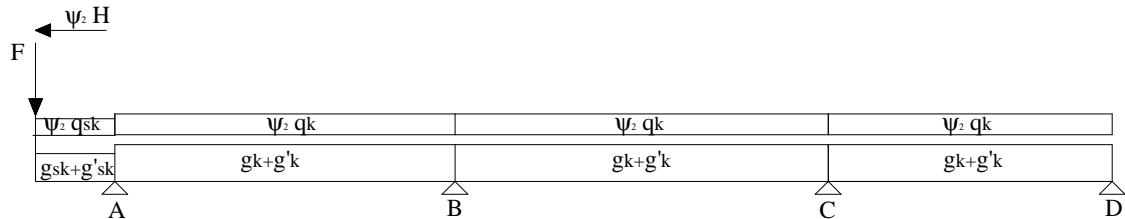
$$p_2 = 6.42 \text{ kN/m}$$

$$L_2 = 5.65 \text{ m}$$

$$X_C^{(Q,P)} = 15.76 \text{ kNm}$$

$$p_3 = 6.42 \text{ kN/m}$$

$$L_3 = 4.30 \text{ m}$$



$$R_A = 20.70 \text{ kN}$$

$$T_A^{(S)} = -6.81 \text{ kN}$$

$$M_A' = -0.30 \text{ kNm}$$

$$R_B = 37.76 \text{ kN}$$

$$T_A^{(D)} = 13.89 \text{ kN}$$

$$M_A = -4.69 \text{ kNm}$$

$$R_C = 35.15 \text{ kN}$$

$$T_B^{(S)} = -19.17 \text{ kN}$$

$$M_B = -18.31 \text{ kNm}$$

$$R_D = 10.14 \text{ kN}$$

$$T_B^{(D)} = 18.59 \text{ kN}$$

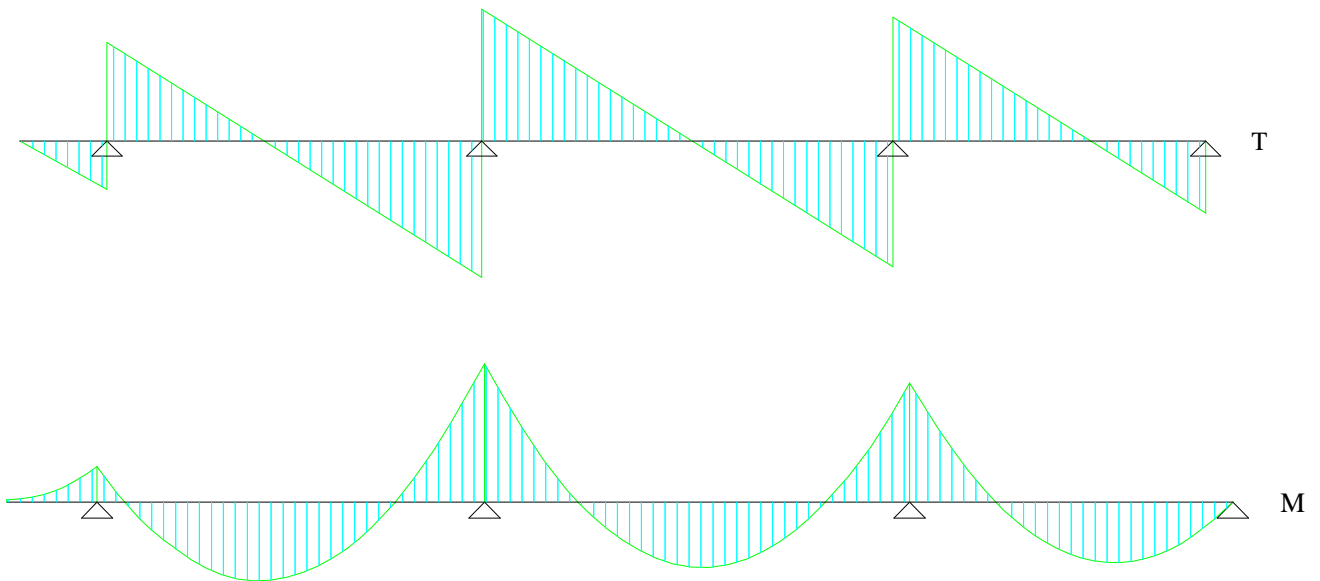
$$M_C = -15.76 \text{ kNm}$$

$$T_C^{(S)} = -17.68 \text{ kN}$$

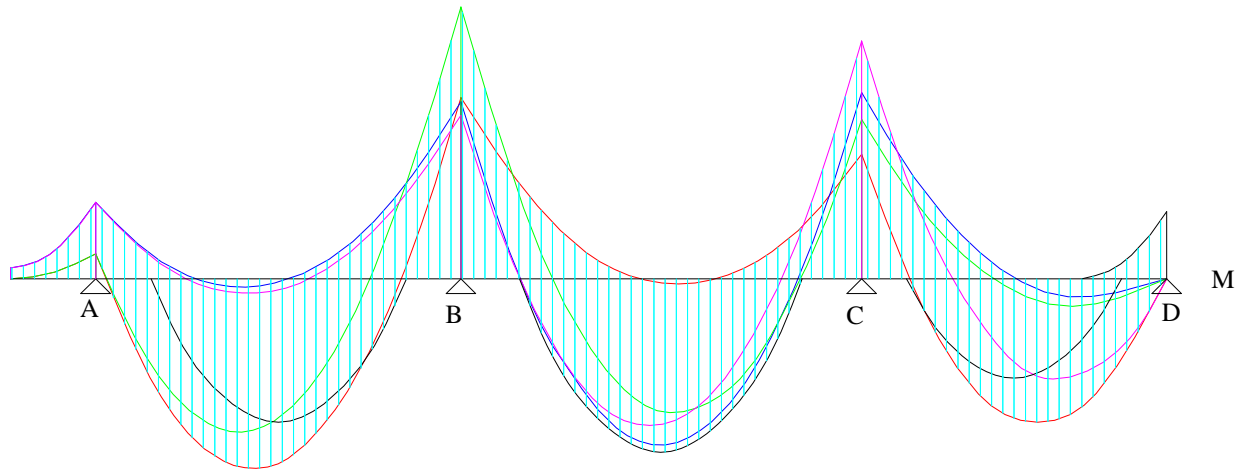
$$M_D = 0$$

$$T_C^{(D)} = 17.47 \text{ kN}$$

$$T_D^{(S)} = -10.14 \text{ kN}$$



Involuppo delle sollecitazioni allo stato limite ultimo



Momenti massimi agli appoggi

$$M_A^{(max)} = -M_A^{(4)} = - 10.75 \text{ kNm}$$

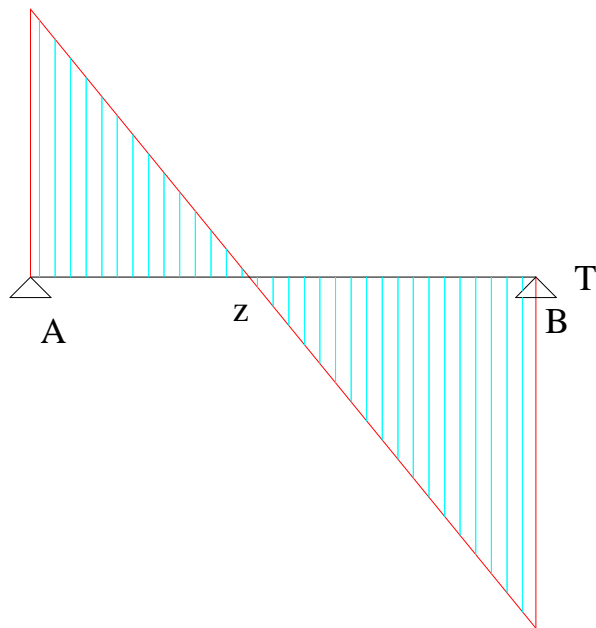
$$M_B^{(max)} = -X_B^{(3)} = - 38.17 \text{ kNm}$$

$$M_C^{(max)} = -X_C^{(4)} = - 33.41 \text{ kNm}$$

$$M_D^{(max)} = -\frac{p_3^{(d)} L_3^2}{24} = - 9.42 \text{ kNm}$$

Momenti massimi in campata

$M_{AB}^{(max)}$ è dato dalla soluzione di carico S.L.U. 1 in corrispondenza del punto in cui il taglio è nullo.



$$T_A^{(D)} = 27.24 \text{ kN}$$

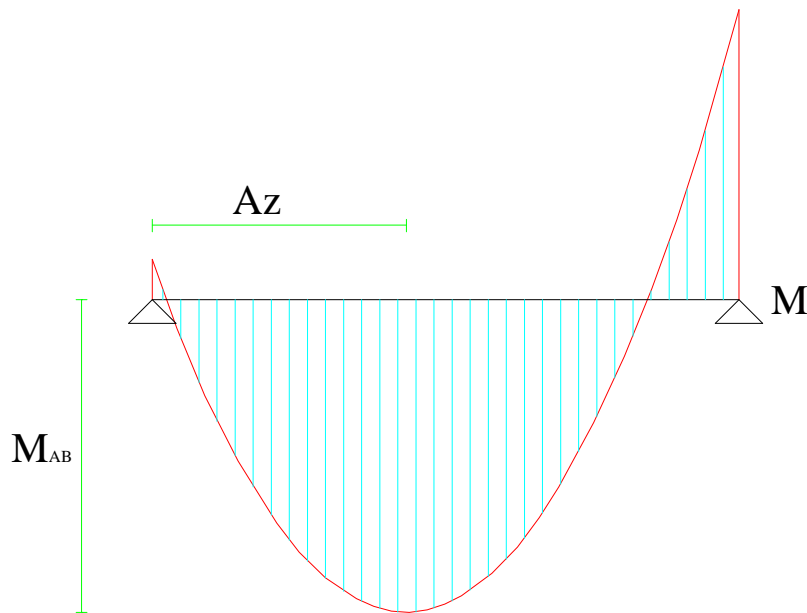
$$T_B^{(S)} = -35.75 \text{ kN}$$

l'equazione del taglio è : $T = Ax + C$

$$\begin{cases} x = 0 \Rightarrow C = 27.24 \\ x = 5.15 \Rightarrow A \cdot 5.15 + 27.24 = -35.75 \Rightarrow A = -12.23 \end{cases}$$

$$-12.23 Az + 27.24 = 0$$

$$Az = \frac{27.24}{12.23} = 2.23 \text{ m}$$



$$M_A = - 3.52 \text{ kNm}$$

$$M_B = - 25.44 \text{ kNm}$$

$$M_{(x)} = \int T dx = \int (-12.23x + 27.24) dx = - 12.23 \frac{x^2}{2} + 27.24 x + \text{Cost}$$

$$x = 0 \Rightarrow \text{Cost} = -3.52$$

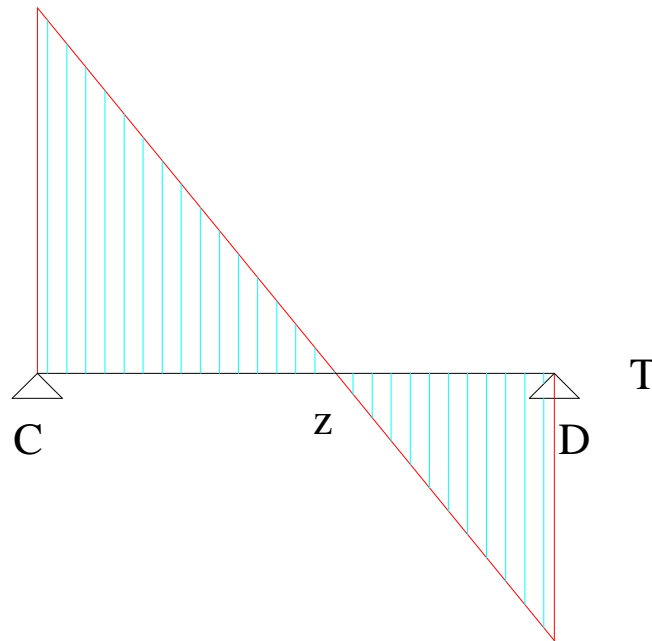
$$-12.23 \frac{x^2}{2} + 27.24 x - 3.52 = 0$$

$$M_{AB}^{(max)} = -12.23 \frac{2.23^2}{2} + 27.24 \cdot 2.23 - 3.52 = 26.82 \text{ kNm}$$

$M_{BC}^{(max)}$ è dato dall'aumento del momento in campata dovuto al semincastro

$$M_{BC}^{(max)} = \frac{P_2^{(max)} L_2^2}{16} = 24.40 \text{ kNm}$$

$M_{CD}^{(max)}$ è dato dalla soluzione di carico S.L.U. 1 in corrispondenza del punto in cui il taglio è nullo.



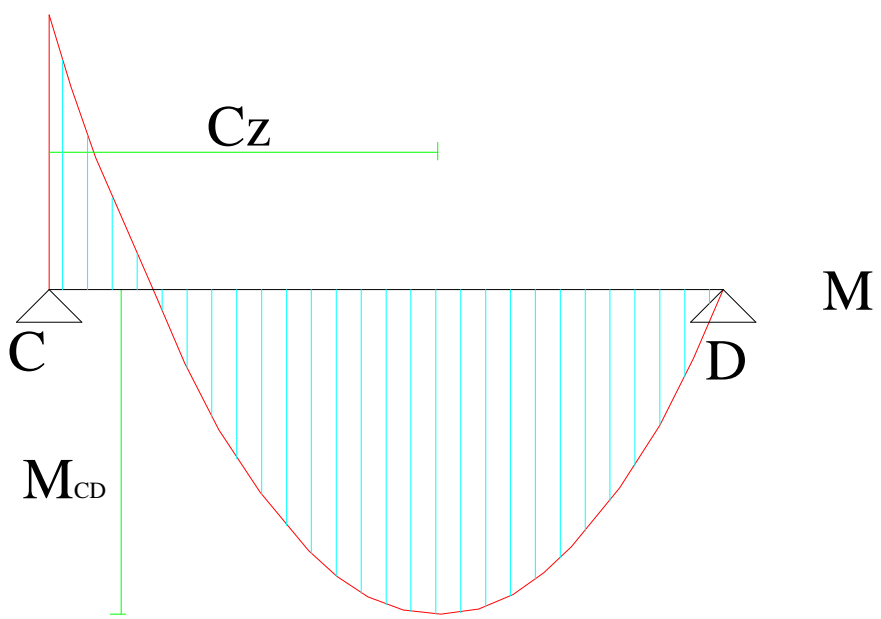
$$T_C^{(D)} = 30.36 \text{ kN} \qquad T_D^{(S)} = -22.22 \text{ kN}$$

l'equazione del taglio è : $T = Ax + C$

$$\begin{cases} x = 0 \Rightarrow C = 30.36 \\ x = 4.30 \Rightarrow A \cdot 4.30 + 30.36 = -22.22 \Rightarrow A = -12.23 \end{cases}$$

$$-12.23 Cz + 30.36 = 0$$

$$Cz = \frac{30.36}{12.23} = 2.48 \text{ m}$$



$$M_C = -17.50 \text{ kNm} \qquad M_D = 0 \text{ kNm}$$

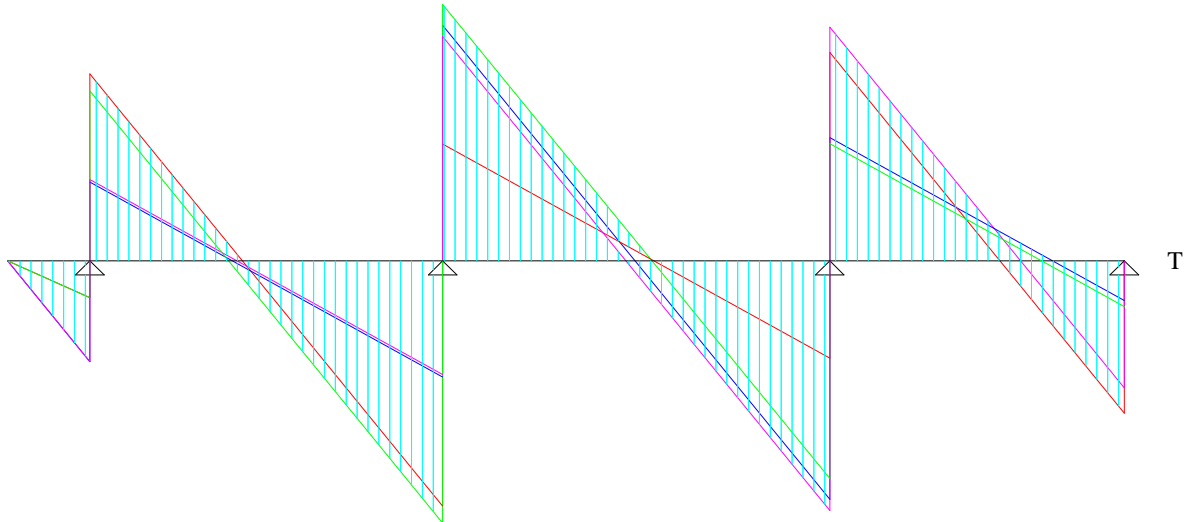
$$M_{(x)} = \int T dx = \int (-12.23x + 30.36) dx = -12.23 \frac{x^2}{2} + 30.36 x + Cost$$

$$x = 0 \Rightarrow \text{Cost} = -17.50$$

$$-12.23 \frac{x^2}{2} + 30.36 x - 17.50 = 0$$

$$M_{CD}^{(max)} = -12.23 \frac{2.48^2}{2} + 30.36 \cdot 2.48 - 17.50 = \mathbf{20.18 \text{ kNm}}$$

Inviluppo del taglio allo stato limite ultimo



$$T_A = \begin{bmatrix} -14.72 \text{ kNm} \\ 27.24 \text{ kNm} \end{bmatrix}$$

$$T_B = \begin{bmatrix} -38.22 \text{ kNm} \\ 37.34 \text{ kNm} \end{bmatrix}$$

$$T_C = \begin{bmatrix} -36.41 \text{ kNm} \\ 34.05 \text{ kNm} \end{bmatrix}$$

$$T_D = 22.22 \text{ kNm}$$

Progetto armatura

$$\text{FeB38K} \Rightarrow f_{sk} = \mathbf{375 \text{ MPa}}$$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{375}{1.15} = \mathbf{326.09 \text{ MPa}}$$

$$R_{ck} = 25 \text{ MPa} \quad f_{ck} = 0.83 R_{ck} = 20,75 \text{ MPa} \quad \Rightarrow$$

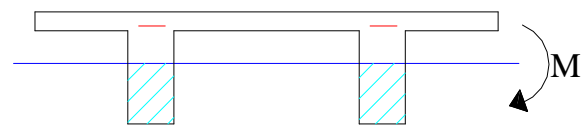
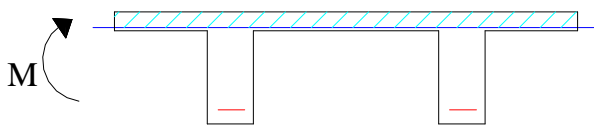
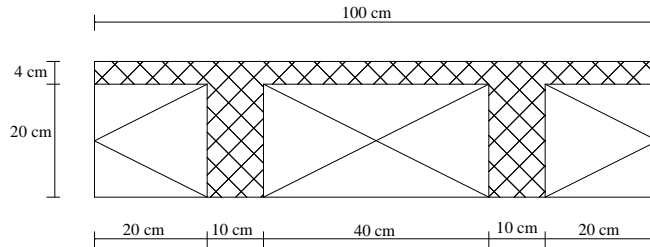
$$f_{ctm} = 0.27 R_{ck}^{(2/3)} = 2.31 \text{ MPa}$$

$$f'_{cd} = \frac{0.83 \cdot 0.85 \cdot R_{ck}}{\gamma_c} = \mathbf{11 \text{ MPa}}$$

$$f_{ctk} = 0.7 f_{ctm} = 1.62 \text{ MPa}$$

$$f_{ctd} = \frac{f_{ctk}}{\gamma_c} = \frac{1.62}{1.6} = \mathbf{1.01 \text{ MPa}}$$

Si progetta un solaio a semplice armatura quindi $\rho = \frac{As'}{As} = 0$



Campate centrali

Sbalzo

h = 24 cm d' = 3 cm

h = 20 cm d' = 3 cm

Utilizzando le formule di progetto

$$r_u = \frac{h}{\sqrt{\frac{Mu}{b}}}$$

$$As = \frac{Mu}{\zeta f_{sd} h}$$

e leggendo i valori corrispondenti di ζ e ξ si ottiene la seguente tabella

sezioni	M [kNm]	b [cm]	r_u	ζ	ξ	As [cm ²]	As,tr [cm ²]	Armature	As [cm ²]
A	-10.75	20	0.327	0.843	0.150	1.63	0.82	2 $\Phi 12$	2.26
AB	26.82	100	0.463	0.900	0.100	3.81	1.91	2 $\Phi 12$	2.26
	-	20	-	-	-	-	-	-	-
B	-38.17	60	0.301	0.835	0.169	5.84	2.92	3 $\Phi 12$	3.39
BC	24.40	100	0.486	0.900	0.100	3.47	1.74	2 $\Phi 12$	2.26
	-	20	-	-	-	-	-	-	-
C	-33.41	20	0.186	0.712	0.450	6.00	3.00	3 $\Phi 12$	3.39
CD	20.18	100	0.534	0.900	0.100	2.87	1.44	2 $\Phi 12$	2.26
	-	20	-	-	-	-	-	-	-
D	-9.42	20	0.350	0.849	0.138	1.42	0.71	1 $\Phi 12$	1.13

Nell'appoggio in B si considera una base di 60 cm al fine di ottenere un armatura di 3 $\Phi 12$; ciò è possibile in quanto in realtà la base è 100 cm perché è presente la trave.

Considerando il taglio si ha:

$$V_{Rd1} = 0.25 b_w d f_{ctd} r (1 + 50 \rho_1) \delta > T$$

$$\delta = 1 \text{ (elemento inflesso)}$$

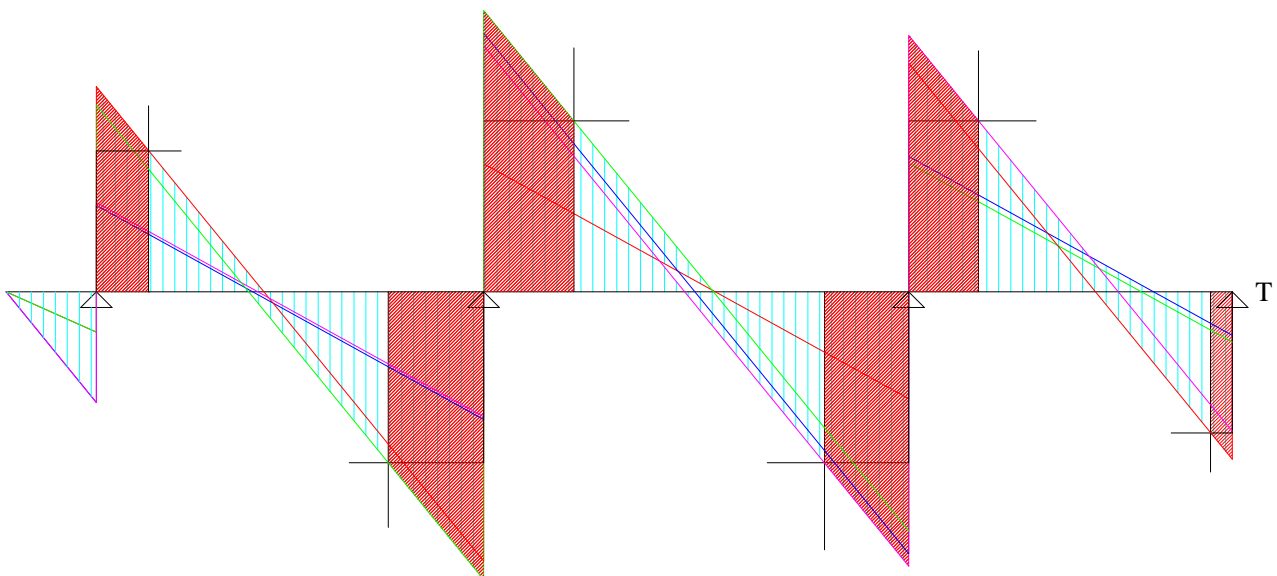
$$r = 1.6 - d > 1.00$$

$$\rho_1 = \frac{A_s}{bd}$$

sezioni	T [kN]	b [cm]	A _{s1} [cm ²]	ρ ₁	V _{Rd1} [KN]	z _f [m]
A	-14.72	20	2.26	0.0066	16.33	0.00
	27.24	20		0.0054	18.72	0.70
B	-38.22	20	4.52	0.0108	22.70	1.27
	37.34	20				1.20
C	-36.41	20	4.52	0.0108	22.70	1.12
	34.05	20				0.93
D	22.22	20	2.26	0.0054	18.72	0.29

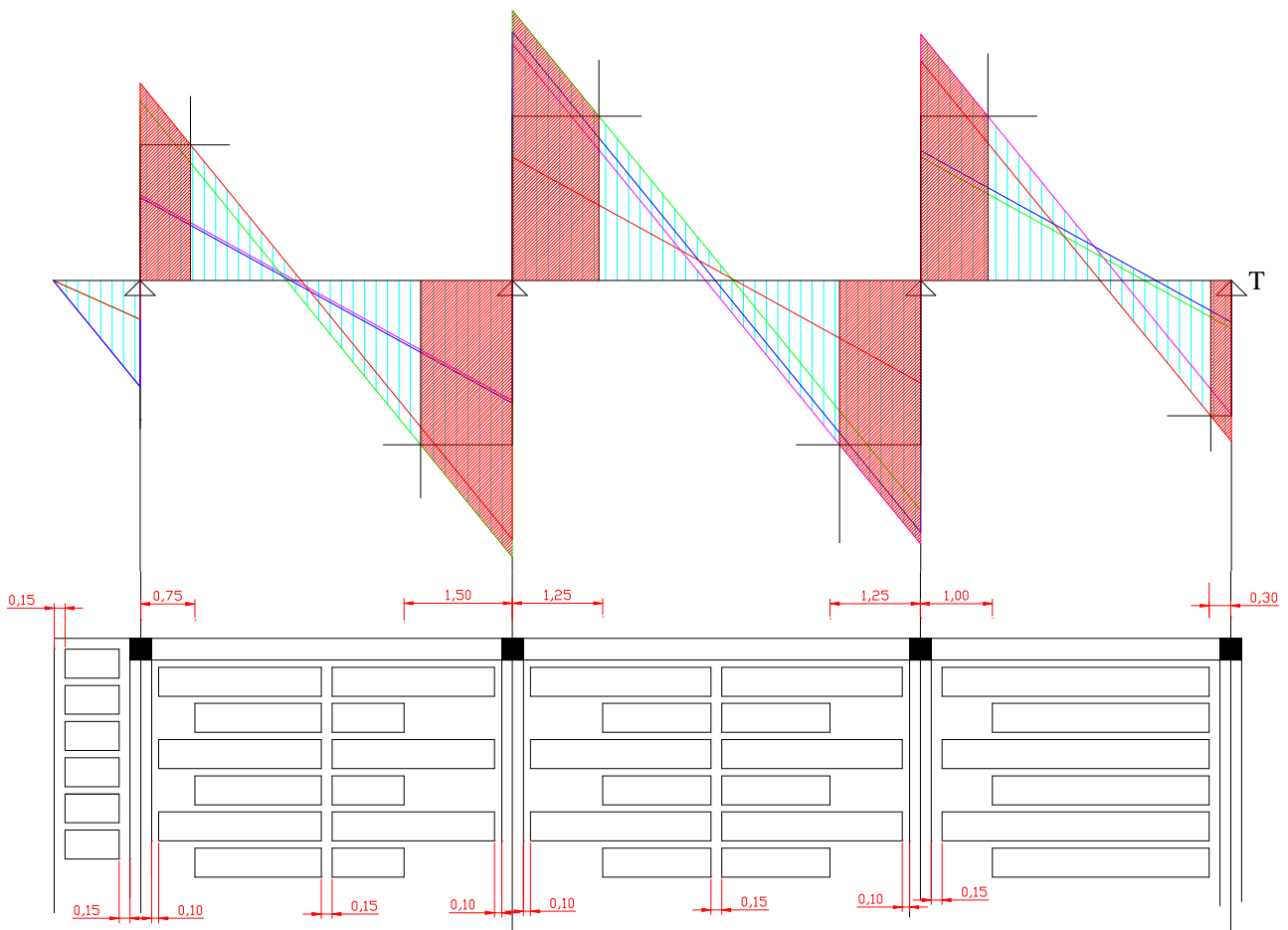
Nelle sezioni B e C si considera un A_s tesa data da due Φ12 per evitare di avere una fascia semipiena eccessivamente grande, ricordando di predisporre un ferro in più a trazione se ne è presente solo uno.

Nei casi in cui V_{Rd1} < T la distanza z_f in cui si ha questa condizione è: $z_f = \frac{|T_i| - V_{Rd1}}{P_d}$



Tratteggiato in rosso vi sono le zone in cui non è soddisfatta la verifica a taglio.

Si vanno quindi a definire delle fascie piene e semipiene allo scopo di aumentare la base per una maggiore resistenza a taglio di tali sezioni.



Determinazione dell'armatura

Considerando le armature inserite in ogni campata, si determina il momento resistente considerando $\zeta = 0.85$

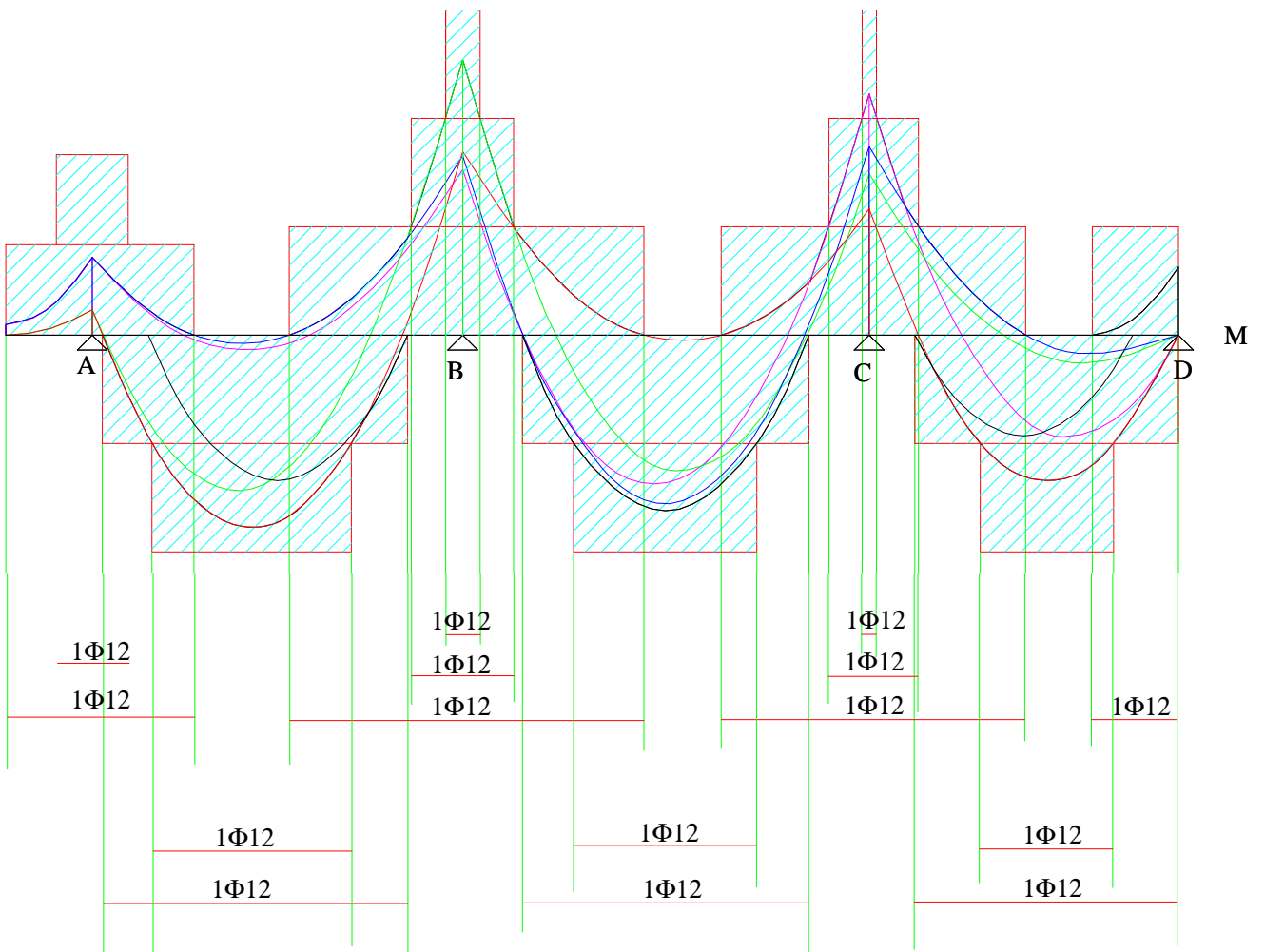
$$M_{Rd} = 0.85 h f_{sd} A_s$$

Tale momento dovrà essere maggiore in ogni punto al momento sollecitante M_{sd} , quindi il diagramma del momento sollecitante dovrà risultare interno al diagramma del momento resistente.

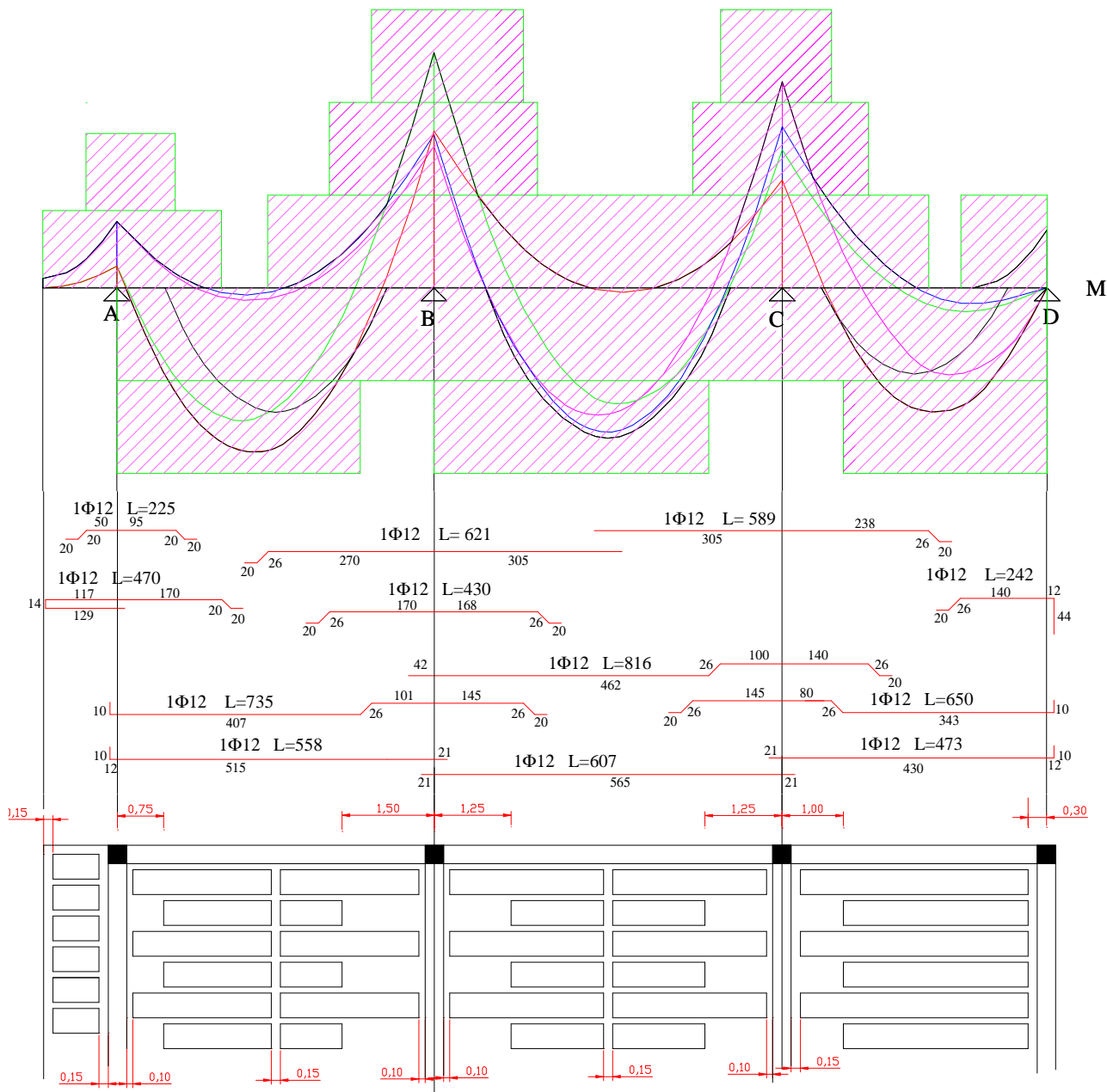
$$\Phi 12 \Rightarrow M_{Rd} = 0.85 \times 200 \times 326 \times 113 \times 2 = 12524920 \text{ Nmm} = \underline{12.52 \text{ kNm}} \text{ sbalzo}$$

$$\Phi 12 \Rightarrow M_{Rd} = 0.85 \times 240 \times 326 \times 113 \times 2 = 15029904 \text{ Nmm} = \underline{15.03 \text{ kNm}}$$

Primaria disposizione delle armature per travetto



Si va quindi a definire la distinta delle armature



Si va a verificare che in appoggio risulti $A_{s_{min}} \geq \frac{V_{sd}}{f_{sd}}$

e per l'armatura inferiore si abbia $A_s > 0.07 h$

Tali verifiche risultano soddisfatte per ogni punto del solaio.

Verifica allo stato limite ultimo

Tale verifica consiste nel determinare il momento resistente M_{Rd} e il taglio resistente V_{Rd} in ogni sezione significativa e confrontarli rispettivamente con il momento e il taglio sollecitante M_{sd} e V_{sd} , verificando che sussistano le seguenti condizioni:

$$|M_{Rd}| \geq |M_{sd}|$$

$$|V_{Rd}| \geq |V_{sd}|$$

con $\Psi = 0.8$ $\lambda = 0.4$

Per basi di 60 e 100 cm si considera a semplice armatura a vantaggio di sicurezza

$$y_c = \frac{A_s f_{sd}}{\Psi b f'_{cd}}$$

$$M_{Rd} = \Psi b y_c f'_{cd} (h - d' - \lambda y_c)$$

Per base di 20 cm si fa la verifica a doppia armatura

$$y_c = \frac{A_s f_{sd} - A' s f_{sd}}{\Psi b f'_{cd}}$$

$$y_{2.2''} = d' + \frac{\frac{f_{sd}}{E_s}}{0.01 + \frac{f_{sd}}{E_s}} \cdot (d - d')$$

$$y_{3.4} = \frac{0.0035}{\frac{f_{sd}}{E_s} + 0.0035} \cdot d$$

Se $y_{2.2''} < y_c < y_{3.4}$

$$M_{Rd} = \Psi b y_c f'_{cd} \left(\frac{h}{2} - \lambda y_c \right) + A' s f_{sd} \left(\frac{h}{2} - d' \right) + A_s f_{sd} \left(\frac{h}{2} - d' \right)$$

Se $y_c < y_{2.2''}$

y_c si ricava dalla risoluzione della seguente equazione di secondo grado

$$0.8b f'_{cd} y_c d - 0.8b f'_{cd} y_c^2 + (A' s E_s 0.01) y_c - (A' s E_s 0.01) d' - A_s f_{sd} d + A_s f_{sd} y_c = 0$$

$$M_{Rd} = 0.8b f'_{cd} y_c \left(\frac{h}{2} - \lambda y_c \right) + A' s E_s \frac{0.01(y_c - d')}{d - y_c} \left(\frac{h}{2} - d' \right) + A_s f_{sd} \left(\frac{h}{2} - d' \right)$$

Se $y_c > y_{3.4}$

y_c si ricava dalla risoluzione della seguente equazione di secondo grado

$$0.8b f'_{cd} y_c^2 + (A' s f_{sd} + A_s E_s 0.0035) y_c - A' s E_s 0.0035 d = 0$$

$$M_{Rd} = 0.8b f'_{cd} y_c \left(\frac{h}{2} - \lambda y_c \right) + A' s f_{sd} \left(\frac{h}{2} - d' \right) + A_s E_s \frac{0.0035}{y_c} (d - y_c) \left(\frac{h}{2} - d' \right)$$

Per il taglio

$$V_{Rd1} = 0.25 b d f_{ctd} r (1 + 50 \rho_l) \delta$$

$$\rho_l = \frac{A_s}{bd}$$

Verifica allo stato limite di servizio

Tale verifica va condotta determinando l'entità delle tensioni nel calcestruzzo e nell'acciaio; in particolare dovrà risultare:

$$\left. \begin{array}{l} \sigma_c \leq 0.60 f_{ck} \\ \sigma_s \leq 0.70 f_{sk} \end{array} \right\} \text{ per la combinazione rara}$$

$$\left. \begin{array}{l} \sigma_c \leq 0.45 f_{sk} \\ \sigma_s \leq 0.70 f_{sk} \end{array} \right\} \text{ per la combinazione quasi permanente}$$

Ciò equivale a verificare che in entrambe le combinazioni i momenti resistenti del cls e dell'acciaio M_{rc} e M_{rs} relativi alle tensioni limite devono risultare maggiori del momento sollecitante.

$$\text{Combinazione rara} \quad \left\{ \begin{array}{l} y_{c,r} = \frac{n(A_s + A'_s)}{b} \left[-1 + \sqrt{1 + \frac{2b(A_s d + A'_s d')}{n(A_s + A'_s)^2}} \right] \\ M_{rc,r} = \frac{I_n}{y_c} 0.60 f_{ck} \\ M_{rs,r} = \frac{I_n}{n(d - y_c)} 0.70 f_{sk} \end{array} \right.$$

$$\text{Combinazione quasi permanente} \quad \left\{ \begin{array}{l} y_{c,qp} = \frac{n(A_s + A'_s)}{b} \left[-1 + \sqrt{1 + \frac{2b(A_s d + A'_s d')}{n(A_s + A'_s)^2}} \right] \\ M_{rc,qp} = \frac{I_n}{y_c} 0.45 f_{ck} \\ M_{rs,qp} = \frac{I_n}{n(d - y_c)} 0.70 f_{sk} \end{array} \right.$$

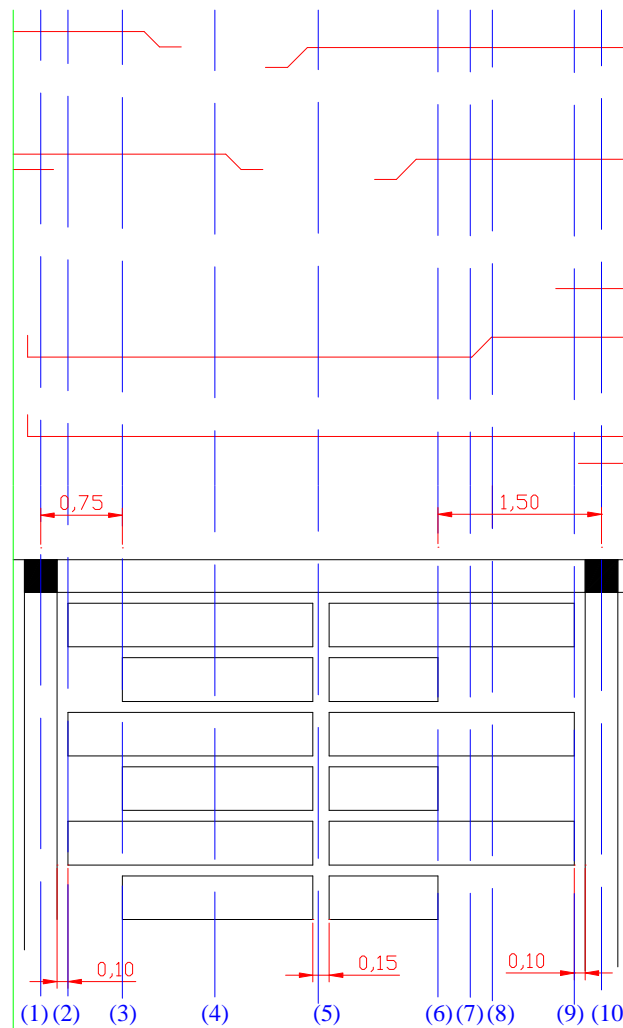
$$I_n = \frac{b y_c^3}{3} + n A'_s (y_c - d')^2 + n A_s (d - y_c)^2$$

$n = 15$ per normativa

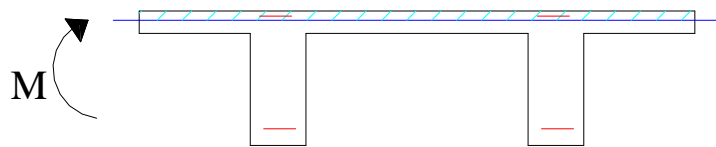
Nel caso del momento positivo se $y_c > 40$ mm la sezione non è più rettangolare ma a T ; in questo caso nei calcoli si trascura il contributo dell'anima a compressione in quanto di piccole dimensioni, il tutto a vantaggio di sicurezza.

$$\text{In momento positivo se } y_c > 40 \text{ mm} \quad \left\{ \begin{array}{l} y_c = \frac{\frac{b s^2}{2} + n A'_s d' + n A_s d}{b s + n A'_s + n A_s} \\ I_n = \frac{b y_c^3}{3} - \frac{b (y_c - s)^3}{3} + n A'_s (y_c - d')^2 + n A_s (d - y_c)^2 \end{array} \right.$$

Campata AB

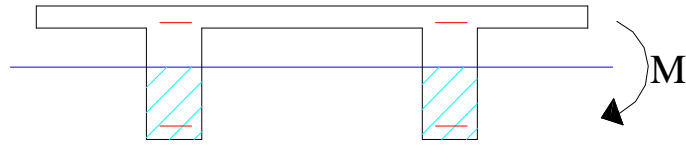


Momento positivo (b = 100)



sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	y _c [mm]	M _{Rd} [kNm]	y _c [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	240	452	452	16,74	29,95	50,38	2,176*10 ⁸	53,77	23,86	40,33	23,86
2	0,25	1000	240	452	452	16,74	29,95	50,38	2,176*10 ⁸	53,77	23,86	40,33	23,86
3	0,75	1000	240	226	452	16,74	29,95	49,05	2,162*10 ⁸	54,88	23,51	41,16	23,51
4	1,60	1000	240	0	452	16,74	29,95	47,54	2,146*10 ⁸	56,20	23,31	42,15	23,31
5	2,55	1000	240	226	452	16,74	29,95	46,35	2,156*10 ⁸	57,91	23,05	43,43	23,05
6	3,65	1000	240	452	452	16,74	29,95	45,32	2,164*10 ⁸	59,45	23,00	44,59	23,00
7	3,95	1000	240	452	226	8,37	15,23	33,80	1,182*10 ⁸	43,54	11,74	32,65	11,74
8	4,14	1000	240	678	226	8,37	15,23	33,53	1,183*10 ⁸	43,93	11,73	32,94	11,73
9	4,9	1000	240	678	226	8,37	15,23	33,53	1,183*10 ⁸	43,93	11,73	32,94	11,73

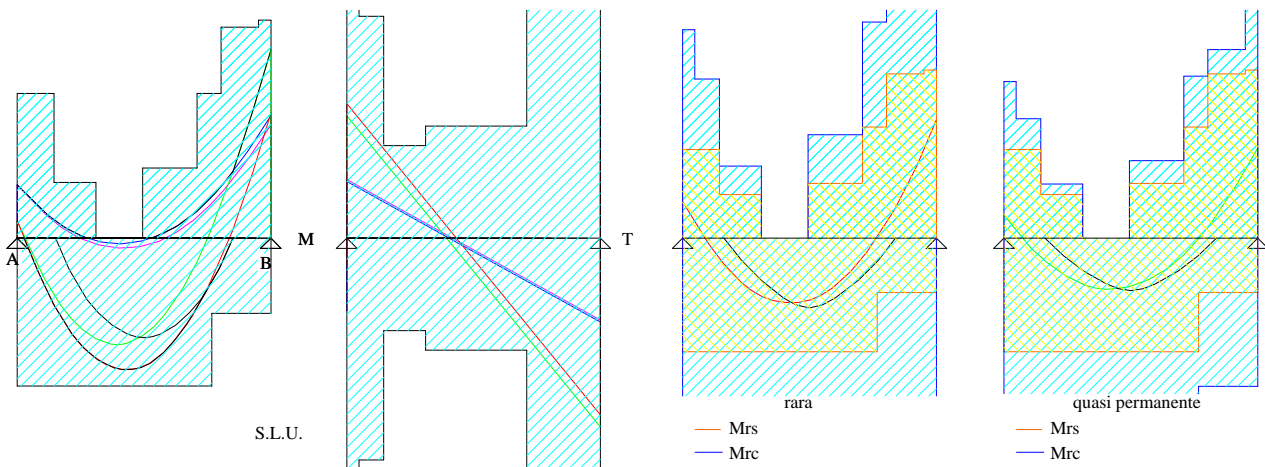
Momento negativo (b variabile)



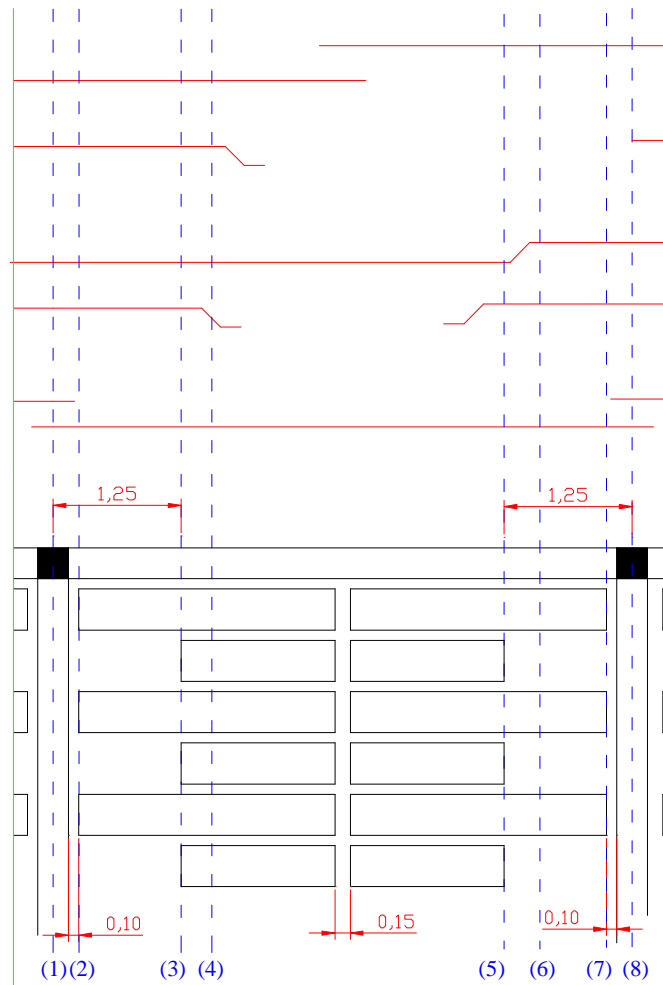
sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	y _c [mm]	M _{Rd} [kNm]	y _c [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	240	452	452	16.74	29.95	40.25	1.366*10 ⁸	42.25	18.42	31.69	18.42
2	0,25	600	240	452	452	27.91	29.30	48.33	1.252*10 ⁸	32.25	18.01	24.19	18.01
3	0,75	200	240	452	226	32.41	11.28	51.05	5.984*10 ⁷	14.59	8.80	10.95	8.80
4	1,60	200	240	452	0	----	-----	-----	-----	-----	-----	-----	-----
5	2,55	200	240	452	226	32.93	14.24	57,49	9,664*10 ⁷	20,93	11,09	15,70	11,09
6	3,65	600	240	452	452	27.91	29.30	54.44	2.004*10 ⁸	45.83	22.54	34.37	22.54
7	3.95	600	240	226	452	27.91	29.30	56.35	1.982*10 ⁸	43.79	22.57	32.84	22.57
8	4,14	600	240	226	678	41.86	42.71	66.67	2.727*10 ⁸	50.92	33.29	38.19	33.29
9	4.9	1000	240	226	678	25.12	44.19	54.69	3.019*10 ⁸	68.73	34.02	51.54	34.02

Taglio

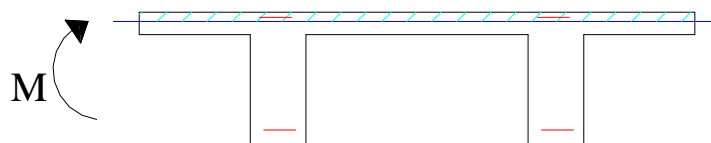
sezioni	z [m]	b [mm]	h [mm]	As _t [mm ²]	ρ _l	V _{Rd1} [kN]	V _{Sd} [kN]
1	0	1000	240	452	0,0027	69,67	27.24
2	0,25	600	240	452	0,0044	44,93	24.18
3	0,75	200	240	226	0,0054	18.72	18,07
4	1,60	200	240	452	0,0108	22,70	7,67
5	2,55	200	240	452	0,0108	22,70	6,42
6	3,65	600	240	452	0,0036	52,18	19,88
7	3.95	600	240	226	0,0018	48,20	23.54
8	4,14	600	240	678	0,0054	56,16	25.87
9	4.9	1000	240	678	0,0032	85,50	35.16



Campata BC

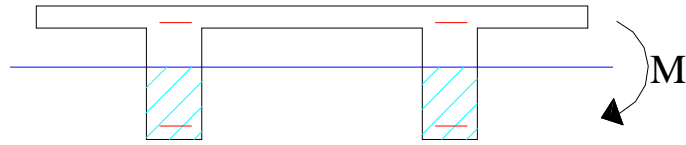


Momento positivo (b = 100 cm)



sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	y _c [mm]	M _{Rd} [kNm]	y _c [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	240	678	452	16,74	29,95	44,25	2,172*10 ⁸	61,11	22,93	45,83	22,93
2	0,25	1000	240	678	452	16,74	29,95	44,25	2,172*10 ⁸	61,11	22,93	45,83	22,93
3	1,25	1000	240	452	452	16,74	29,95	45,32	2,164*10 ⁸	59,45	23,00	44,59	23,00
4	1,55	1000	240	226	452	16,74	29,95	46,35	2,156*10 ⁸	57,91	23,05	43,43	23,05
5	4,40	1000	240	452	226	8,37	15,23	33,80	1,182*10 ⁸	43,54	11,74	32,65	11,74
6	4,75	1000	240	678	226	8,37	15,23	33,53	1,183*10 ⁸	43,93	11,73	32,94	11,73
7	5,40	1000	240	678	226	8,37	15,23	33,53	1,183*10 ⁸	43,93	11,73	32,94	11,73

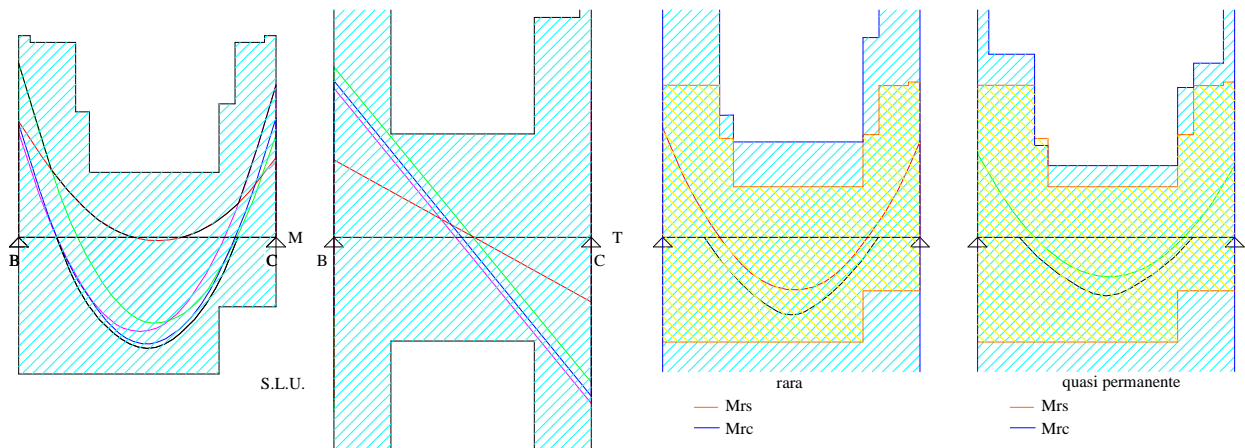
Momento negativo (b variabile)



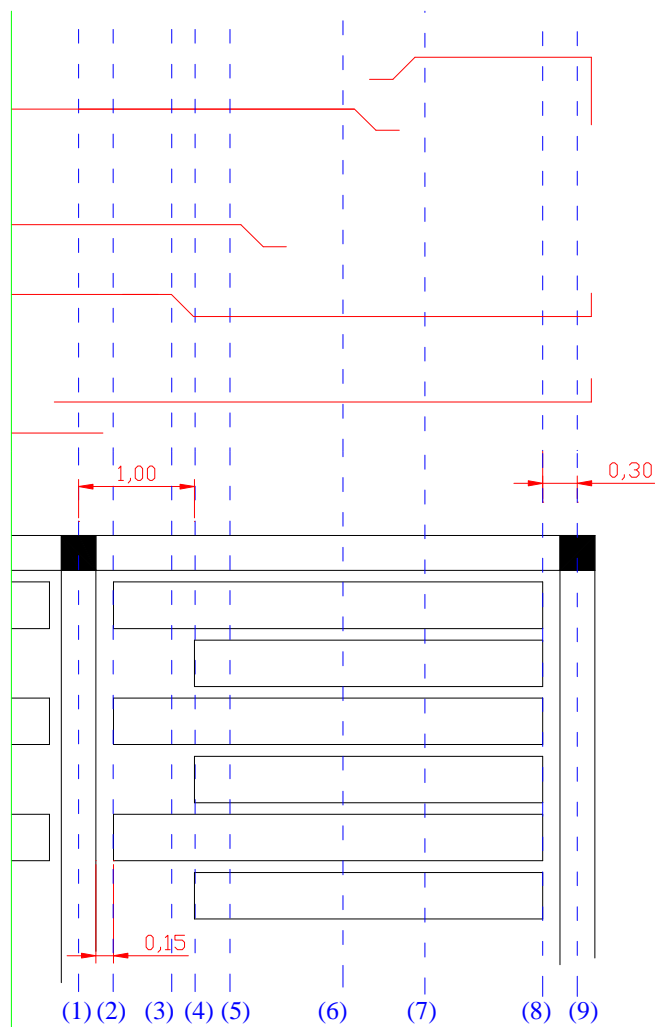
sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	yc [mm]	M _{Rd} [kNm]	yc [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	240	452	678	25,12	44,20	53,52	3,039*10 ⁸	70,69	33,99	53,02	33,99
2	0,25	600	240	452	678	41,86	42,71	64,46	2,770*10 ⁸	53,50	33,31	40,12	33,31
3	1,25	200	240	452	452	42,71	27,49	76,66	1,653*10 ⁸	26,84	21,69	20,13	21,69
4	1,55	200	240	452	226	32,93	14,24	57,49	9,664*10 ⁷	20,93	11,09	15,70	11,09
5	4,40	600	240	226	452	27,91	29,30	56,35	1,982*10 ⁸	43,79	22,57	32,84	22,57
6	4,75	600	240	226	678	41,86	42,71	66,67	2,727*10 ⁸	50,92	33,29	38,19	33,29
7	5,40	1000	240	226	678	25,12	44,19	54,69	3,019*10 ⁸	68,73	34,02	51,54	34,02

Taglio

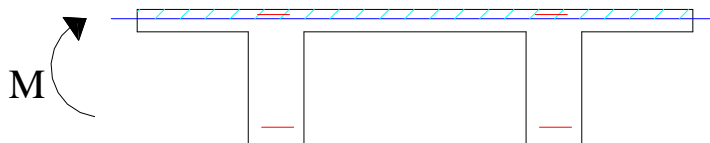
sezioni	z [m]	b [mm]	h [mm]	As _t [mm ²]	ρl	V _{Rd} [kN]	V _{Sd} [kN]
1	0	1000	240	678	0,0032	85,50	37,34
2	0,25	600	240	678	0,0054	56,16	34,28
3	1,25	200	240	452	0,0108	22,70	22,05
4	1,55	200	240	452	0,0108	22,70	18,38
5	4,40	600	240	226	0,0018	48,20	21,12
6	4,75	600	240	226	0,0018	48,20	25,40
7	5,40	1000	240	678	0,0032	85,50	33,35



Campata CD

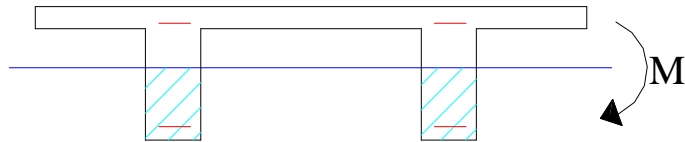


Momento positivo (b = 100 cm)



sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	y _c [mm]	M _{Rd} [kNm]	y _c [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	240	678	226	8,37	15,23	33,53	1,183*10 ⁸	43,93	11,73	32,94	11,73
2	0,30	1000	240	678	226	8,37	15,23	33,53	1,183*10 ⁸	43,93	11,73	32,94	11,73
3	0,80	1000	240	452	226	8,37	15,23	33,80	1,182*10 ⁸	43,54	11,74	32,65	11,74
4	1,00	1000	240	452	452	16,74	29,95	45,32	2,164*10 ⁸	59,45	23,00	44,59	23,00
5	1,30	1000	240	226	452	16,74	29,95	46,35	2,156*10 ⁸	57,91	23,05	43,43	23,05
6	2,30	1000	240	0	452	16,74	29,95	47,54	2,146*10 ⁸	56,20	23,31	42,15	23,31
7	3,00	1000	240	226	452	16,74	29,95	46,35	2,156*10 ⁸	57,91	23,05	43,43	23,05
8	4,00	1000	240	226	452	16,74	29,95	46,35	2,156*10 ⁸	57,91	23,05	43,43	23,05

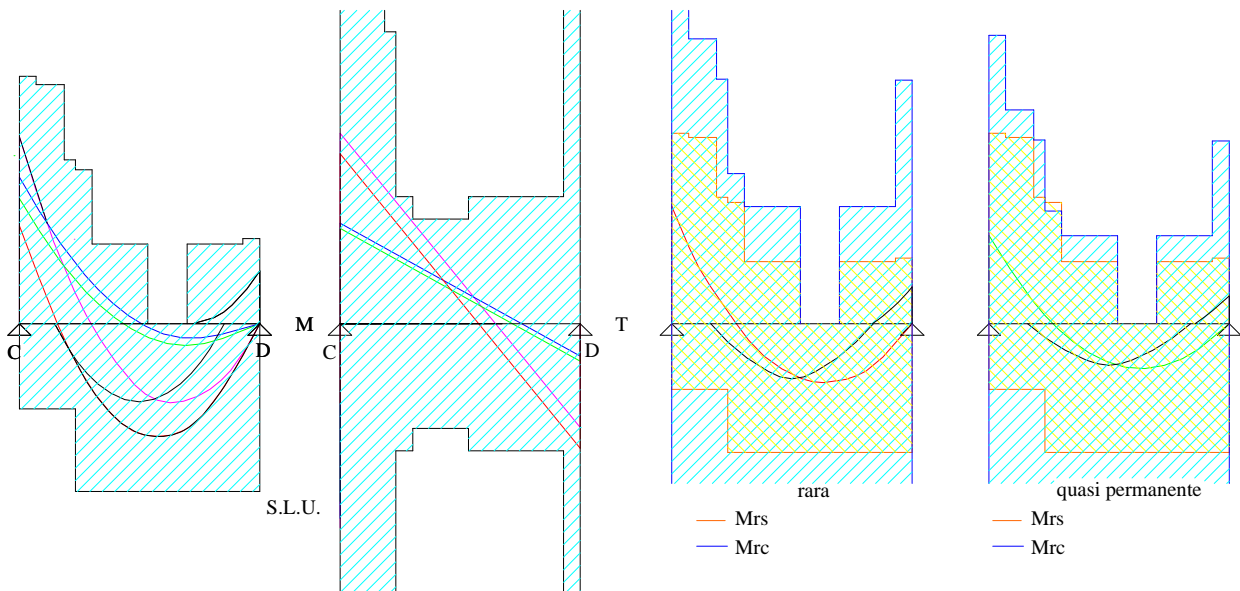
Momento negativo (b variabile)



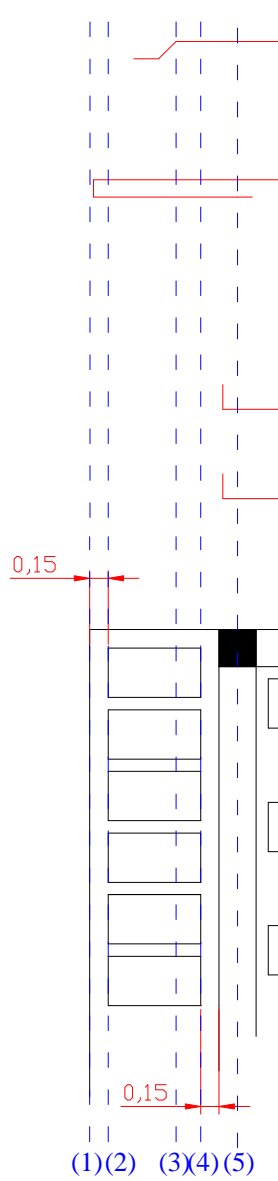
sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	y _c [mm]	M _{Rd} [kNm]	y _c [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	240	226	678	25.12	44.19	54.69	3.019*10 ⁸	68.73	34.02	51.54	34.02
2	0.30	600	240	226	678	41.86	42.71	66.67	2.727*10 ⁸	50.92	33.29	38.19	33.29
3	0.80	600	240	226	452	27.91	29.30	56.35	1.982*10 ⁸	43.79	22.57	32.84	22.57
4	1,00	200	240	452	452	42,71	27,49	76,66	1,653*10 ⁸	26,84	21,69	20,13	21,69
5	1,30	200	240	452	226	32.93	14.24	57,49	9,664*10 ⁷	20,93	11,09	15,70	11,09
6	2,30	200	240	452	0	-----	-----	-----	-----	-----	-----	-----	-----
7	3,00	200	240	452	226	32.93	14.24	57,49	9,664*10 ⁷	20,93	11,09	15,70	11,09
8	4,00	1000	240	452	226	8,37	15,23	33,80	1,182*10 ⁸	43,54	11,74	32,65	11,74

Taglio

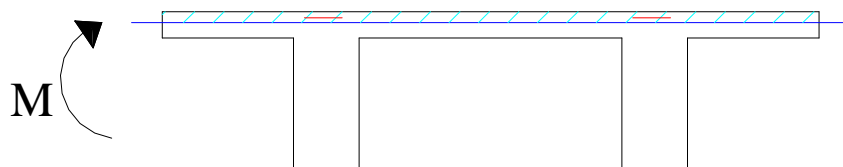
sezioni	z [m]	b [mm]	h [mm]	As _t [mm ²]	ρ _l	V _{Rd} [kN]	V _{Sd} [kN]
1	0	1000	240	678	0,0032	85,50	34,05
2	0.30	600	240	678	0,0054	56,16	30,38
3	0.80	600	240	452	0,0036	52,18	24,27
4	1,00	200	240	452	0,0108	22,70	21,82
5	1,30	200	240	226	0,0054	18,72	18,15
6	2,30	200	240	452	0,0108	22,70	5,92
7	3,00	200	240	452	0,0108	22,70	6,32
8	4,00	1000	240	226	0,0011	77,76	18,55



Sbalzo

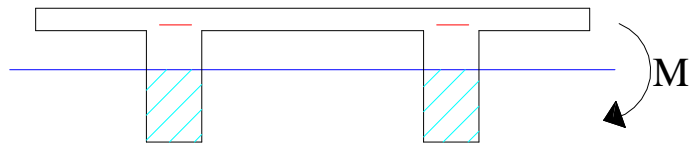


Momento positivo (b = 100 cm)



sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	y _c [mm]	M _{Rd} [kNm]	y _c [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	200	226	0	-----	-----	-----	-----	-----	-----	-----	-----
2	0,15	1000	200	226	0	-----	-----	-----	-----	-----	-----	-----	-----
3	0,70	1000	200	452	0	-----	-----	-----	-----	-----	-----	-----	-----
4	0,90	1000	200	452	0	-----	-----	-----	-----	-----	-----	-----	-----

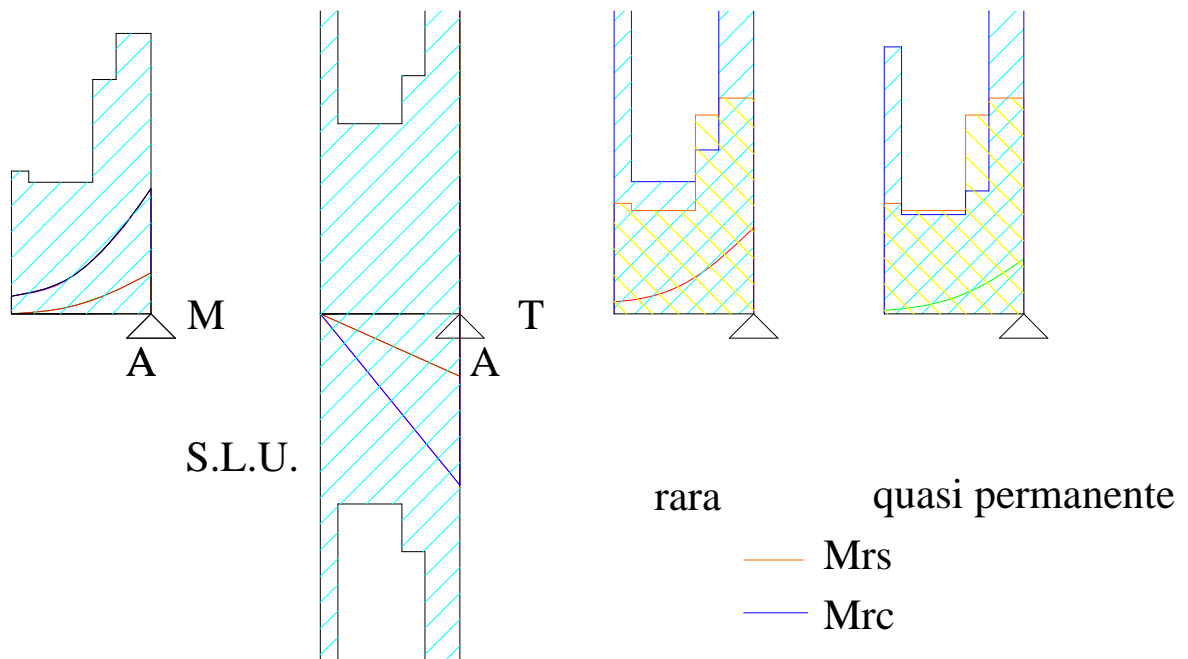
Momento negativo (b variabile)

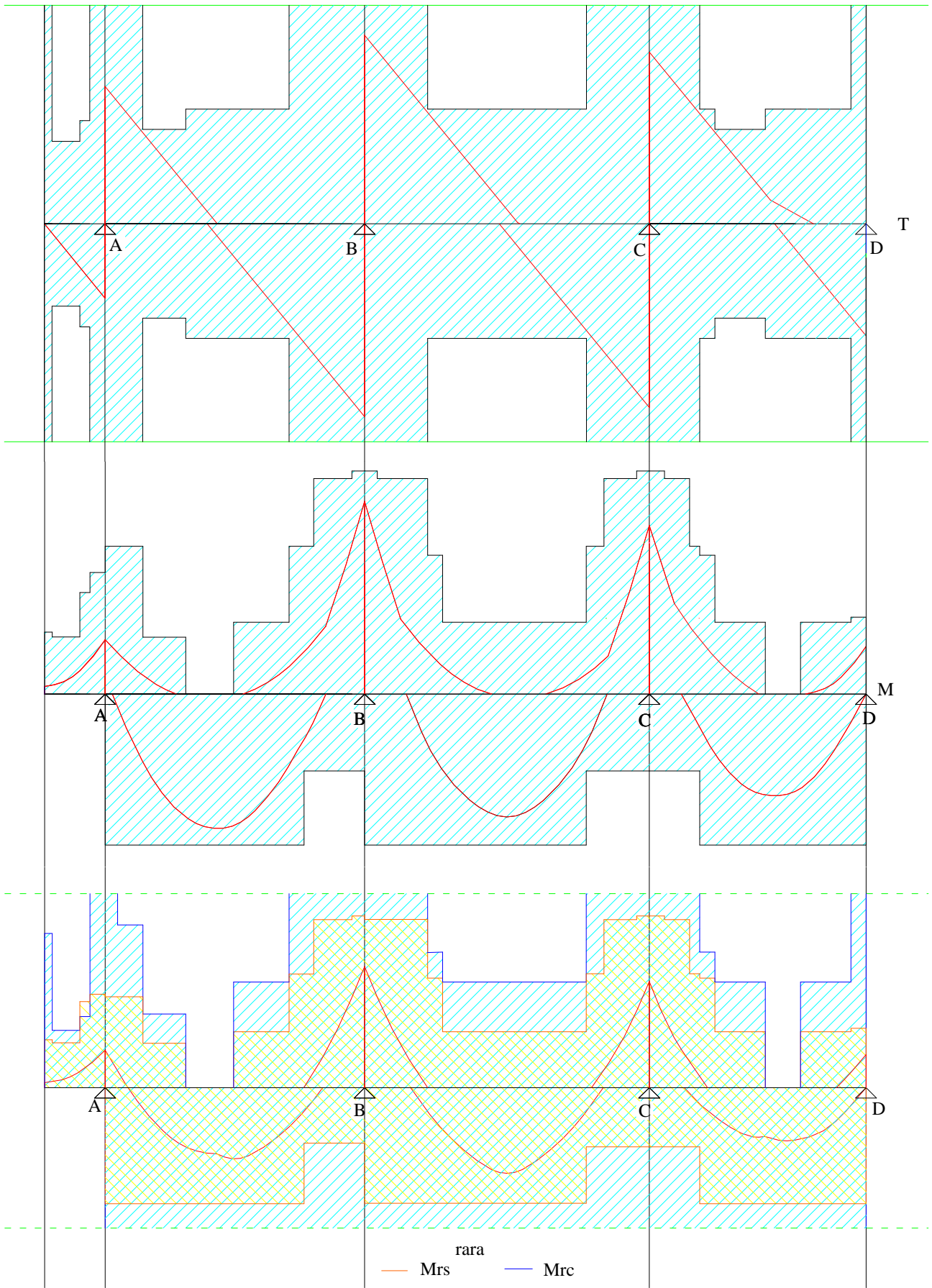


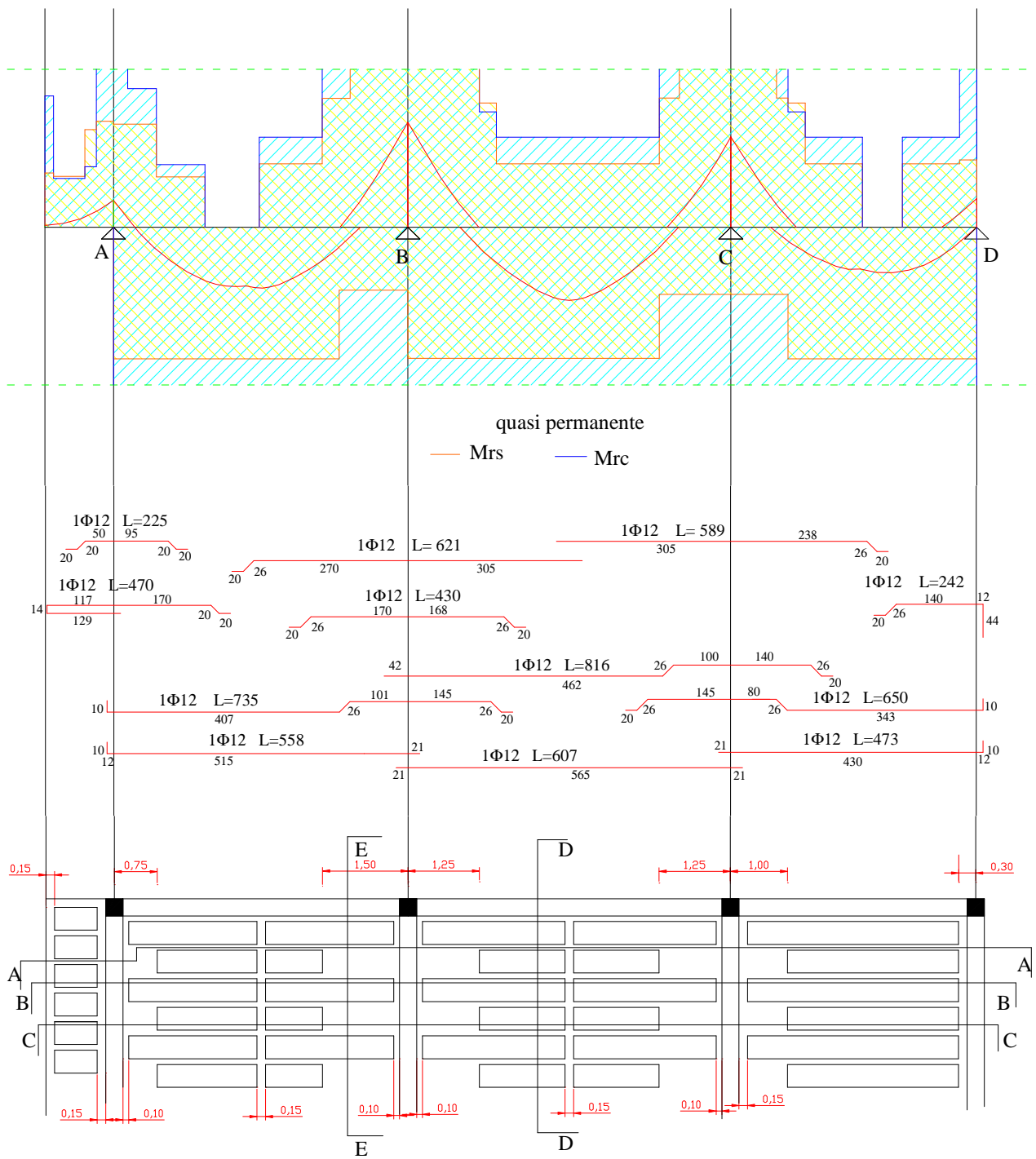
sez	z [m]	b [mm]	h [mm]	A's [mm ²]	As [mm ²]	y _c [mm]	M _{Rd} [kNm]	y _c [mm]	I _n [mm ⁴]	M _{rc,r} [kNm]	M _{rs,r} [kNm]	M _{rc,qp} [kNm]	M _{rs,qp} [kNm]
1	0	1000	200	0	226	8,37	12,27	30,73	7,543*10 ⁷	30,56	9,48	22,92	9,48
2	0,15	200	200	0	226	41,86	11,29	60,83	5,541*10 ⁷	11,34	8,88	8,51	8,88
3	0,7	200	200	0	452	83,72	20,11	78,68	8,901*10 ⁷	14,08	17,06	10,56	17,06
4	0,90	1000	200	0	452	16,74	24,06	41,71	1,358*10 ⁸	40,53	18,52	30,40	18,52

Taglio

sezioni	z [m]	b [mm]	h [mm]	A _{S_{tot}} [mm ²]	ρ _l	V _{Rd} [kN]	V _{Sd} [kN]
1	0	1000	200	226	0,0013	65,37	0
2	0,15	200	200	226	0,0066	16,33	1,84
3	0,7	200	200	452	0,0133	20,44	8,59
4	0,9	1000	200	452	0,0027	69,67	11,04

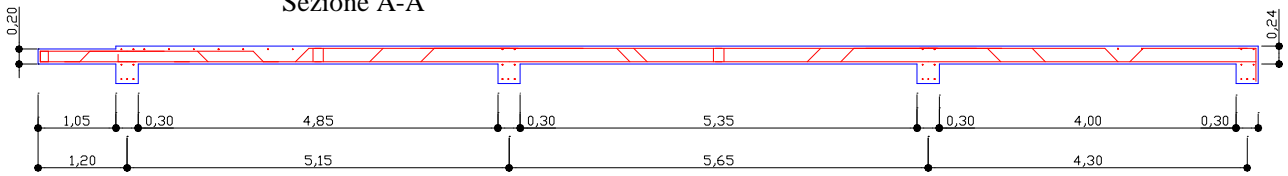




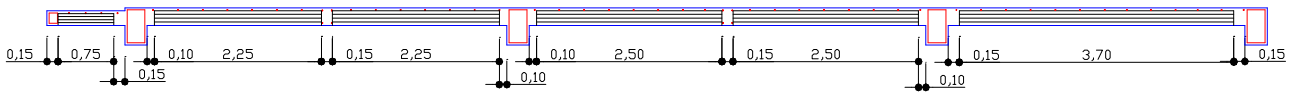


Sezioni

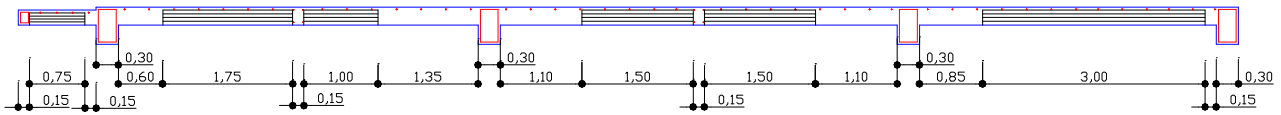
Sezione A-A



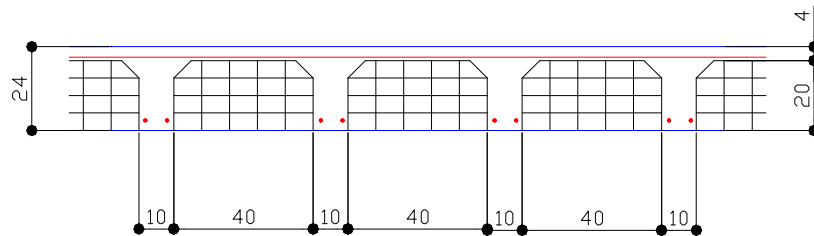
Sezione B-B



Sezione C-C



Sezione D-D



Sezione E-E

