Università degli Studi di Salerno – Facoltà di Ingegneria
Corso di Tecnica delle Costruzioni I – Nuovo Ordinamento
2ª Prova intercorso - 22/07/2008
Anno accademico 2007-2008

Esercizio n. 1 (Punti 8)

Si analizzi la struttura rappresentata nella figura seguente per la quale si assumono i seguenti valori numerici delle grandezze geometrico-meccaniche:

Sezioni

Tratti AB – CD - EG

b=30 cm; h=20+5 *N* [cm]

Pendolo CG

b_p=h_p =20+2 *C* [cm]

Dimensioni

L=1.0+2 (*N*+*C*+*M*)/10 [m]

L_p=2.0 (*N*+*C*)/10 [m]

H=1.0+2 (*N*+*C*)/10 [m]

Azioni

F= 10+2*M* [kN]

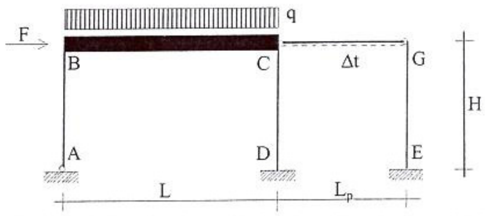
q= 10+2*N* [kN/m]

Δt= *N*+*C*+*M* [°C]

α = *0.00001* [°C^{-1}]

Calcestruzzo

R_{ck}= 25.0 MPa



Si traccino i diagrammi delle caratteristiche della sollecitazione (N,T,M).

N.B.: in questo esercizio e nei seguenti si indica con *N* ed *C* il numero di lettere che costituiscono rispettivamente il nome e cognome del candidato. *M* è l'ultima cifra del numero di matricola.

Esercizio n. 2 (Punti 14)

Per la struttura rappresentata, si traccino i diagrammi delle Caratteristiche della Sollecitazione assumendo:

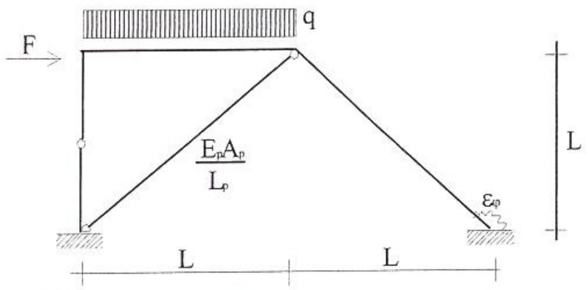
- L= 4.0+0.2 *C* [m];
- F=100+ *N* [kN]
- q=F/L;
- *ritti:*
  - b_r=30 cm;
  - h_r=40 cm;
- *traverso:*
  - b_t=30 cm;
  - h_t=40 + *C* cm;

E_c=28500 MPa

E_p=205000 MPa

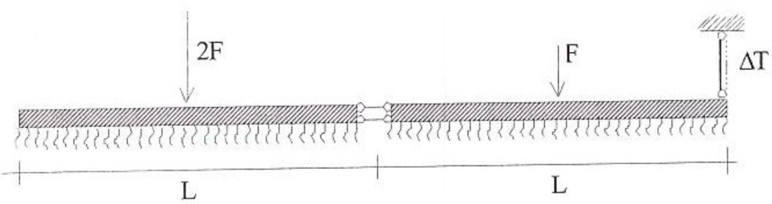
A_p= 700 mm^2

ε_φ=L/(E_c b_r h_r^3)



Esercizio n. 3 (Punti 8)

Ancora con riferimento agli stessi dati del primo esercizio si risolva la trave di fondazione rappresentata nella figura seguente tracciandone i diagrammi delle caratteristiche della sollecitazione (N, T, M):

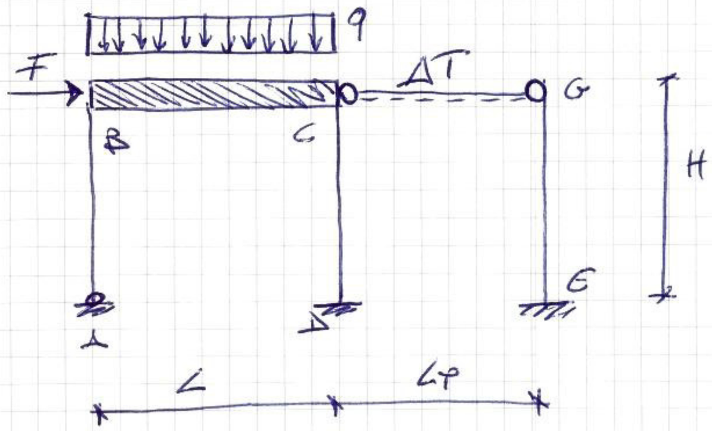


k_0=0.01 N/mm^3

B=80+5*M* [cm]

(pendolo inestensibile)

Esercizio n.1 del 22/07/2008



Dati:

sezioni AB-CD-EG
$b = 30\,cm$
$h = 50\,cm$

pendolo
$b = 30\,cm$
$h = 30\,cm$

dimensioni
$L = 5\,m$
$L_p = 4\,m$
$H = 3{,}5\,m$

azioni
$F = 25\,kN$
$q = 18\,kN/m$
$\Delta T = 15°C$

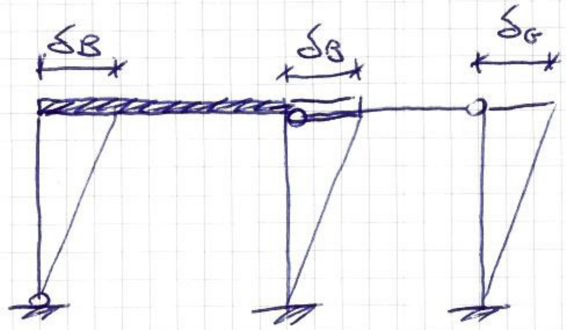
caratteristiche dei materiali
$R_{ck} = 25\,MPa$ (C20/25)
$\alpha = 0{,}00001\ °C^{-1}$

- Classificazione della struttura:

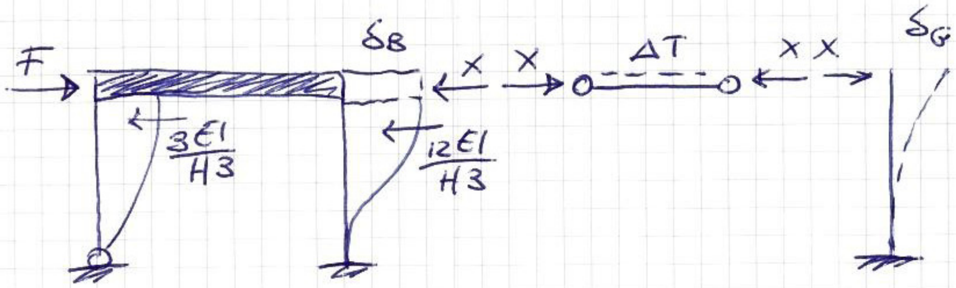
$$3t - 2c = 3 \cdot 4 - 2 \cdot 5 = 2$$
$$\Downarrow$$

N.B. la presenza del pendolo rende dipendenti i 2 spost. incogniti quindi la struttura è a un solo nodo spostabile.

Cinematismo:



Trattandosi di un telaio con traverso rigido e maglie rettangolari, δ_B può essere determinato con un eq. alla traslazione



$$\begin{cases} \delta_G = \dfrac{H^3 X}{3EI} \\ \delta_B - \delta_G = \dfrac{X L_p}{E A_p} - \alpha \Delta T L_p \end{cases}$$

considerando positive le contrazioni

$$\Downarrow$$

$$\delta_B = \frac{X L_p}{E A_p} - \alpha \Delta T L_p + \frac{H^3 X}{3EI}$$

$$X \left(\frac{L_p}{E A_p} + \frac{H^3}{3EI} \right) = \delta_B + \alpha \Delta T L_p$$

$$\Downarrow$$

$$X = \frac{\delta_B + \alpha \Delta T L_p}{\left(\dfrac{L_p}{E A_p} + \dfrac{H^3}{3EI} \right)}$$

Passiamo alla risoluzione del Telaio

$$F - \frac{3EI}{H^3}\delta_B - \frac{12EI}{H^3}\delta_B - X = 0$$

$$\Downarrow$$

$$\boxed{F - \frac{3EI}{H^3}\delta_B - \frac{12EI}{H^3}\delta_B - \frac{\delta_B + \alpha\Delta T L_p}{\frac{L_p}{EA_p} + \frac{H^3}{3EI}} = 0}$$

$$E = 22\,000\left[\frac{f_{cm}}{10}\right]^{0,3}$$

$f_{ck} = 0,83\ R_{ck} = 0,83 \cdot 25 = 20,75$ MPa

$f_{cm} = f_{ck} + 8 = 28,75$ MPa

$E = 30200$ MPa

$$I = \frac{bh^3}{12} = \frac{30 \cdot 50^3}{12} = 3,125 \cdot 10^5\ cm^4 = 3,125 \cdot 10^9\ mm^4$$

$A_p = 300 \cdot 300 = 9 \cdot 10^4\ mm^2$

$$25000 - \frac{3 \cdot 30200 \cdot 3,125 \cdot 10^9}{3500^3}\delta_B - \frac{12 \cdot 30200 \cdot 3,125 \cdot 10^9}{3500^3}\delta_B - \frac{\delta_B + 0,00001 \cdot 15 \cdot 4000}{\frac{4000}{30200 \cdot 9 \cdot 10^4} + \frac{3500^3}{3 \cdot 30200 \cdot 3,125 \cdot 10^9}} = 0$$

$$25000 - 6603,49 \cdot \delta_B - 26413,99\,\delta_B - 6539,94\,\delta_B - 3923,96 = 0$$

$$\Downarrow$$

$$\boxed{\delta_B = 0,5328\ mm}$$

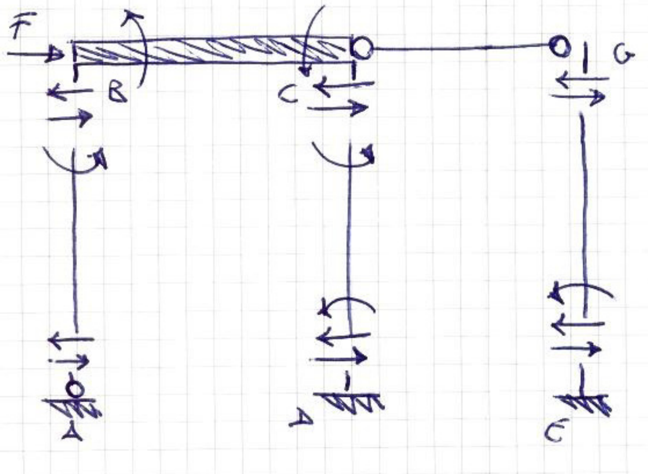
$$X = \frac{\delta_B + \alpha \Delta T L_p}{\frac{L_p}{E A_p} + \frac{H^3}{3EI}} = 7,41\ kN$$

$$\delta_G = \frac{X H^3}{3EI} = 1,13\ mm$$

$$T_{AB} = \frac{3EI}{H^3}\delta_B = 3,52\ kN = T_{BA}$$

$$T_{CD} = \frac{12EI}{H^3}\delta_B = T_{DC} = 14,07\ kN$$

$$T_{EG} = X = 7,41\ kN$$



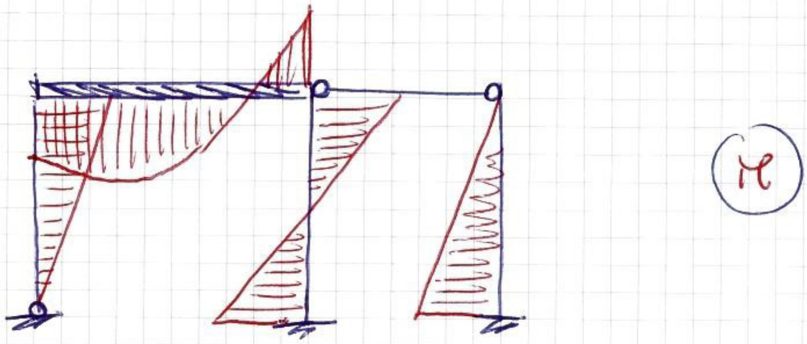
$$M_{BA} = T_{AB} \cdot H = 12,32\ kNm$$

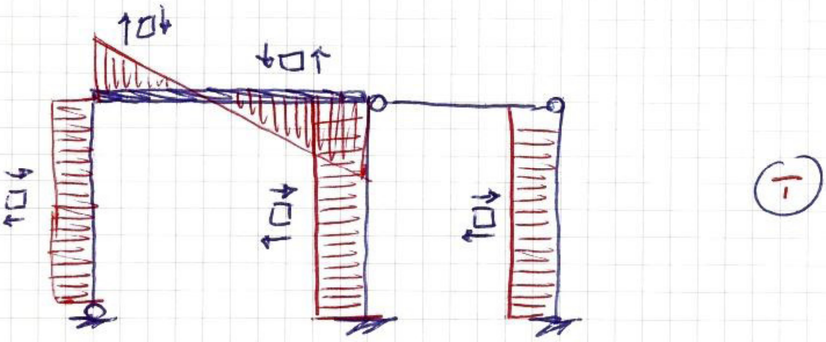
$$M_{EG} = X \cdot H = 25,94\ kNm$$

$$M_{DC} = M_{CD} = T_{CD} \cdot H/2 = 24,62\ kNm$$

$$M_{BC} = -M_{BA} = -12,32\ kNm$$

$$M_{CB} = -M_{CD} = -24,62\ kNm$$

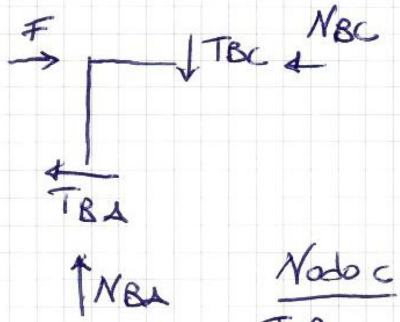




$$T_{BC} = \frac{qL}{2} + \frac{M_{BC} + M_{CB}}{L} = 37{,}61 \ kN$$

$$T_{CB} = -\frac{qL}{2} + \frac{M_{BC} + M_{CB}}{L} = -52{,}32 \ kN$$

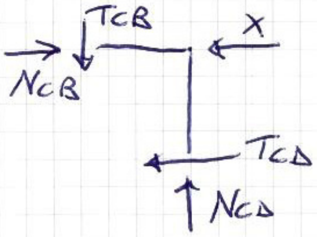
Nodo B



$$N_{BA} = T_{BC} = 37{,}61 \ kN$$

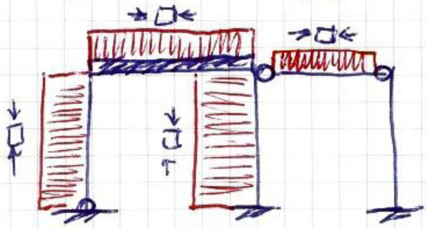
$$N_{BC} = F - T_{BA} = 25 - 3{,}52 = 21{,}48 \ kN$$

Nodo c

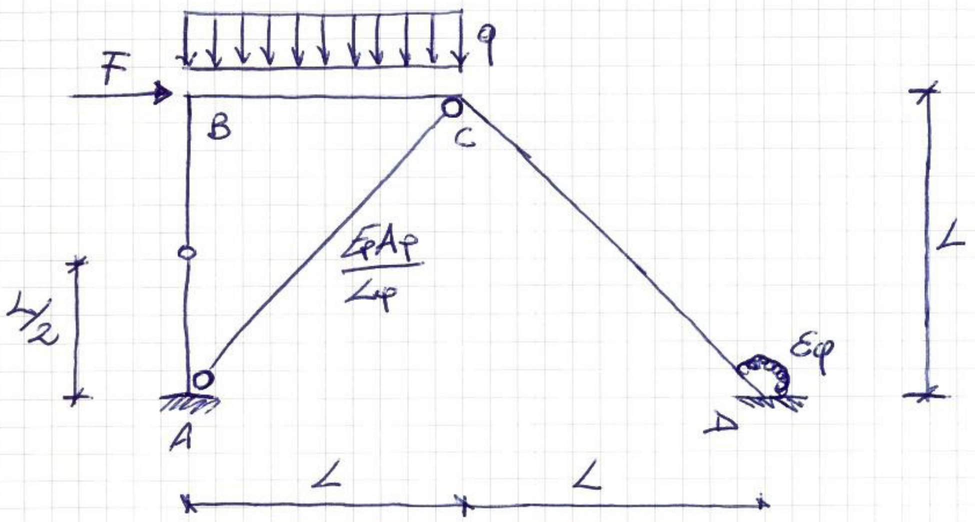


$$N_{CD} = T_{CB} = 52{,}32 \ kN$$

$$N_{CB} = X + T_{CD} = 21{,}48 \ kN = N_{BC}$$



Esercizio n. 3 del 22/07/2008



Dati:

$L = 4{,}00\ m$

$F = 100\ KN$

$q = 20\ KN/m$

$E_c = 28500\ MPa$

$E_f = 205000\ MPa$

$A_f = 700\ mm^2$

-sezioni AB-CD

$b_r = 30\ cm$

$h_r = 40\ cm$

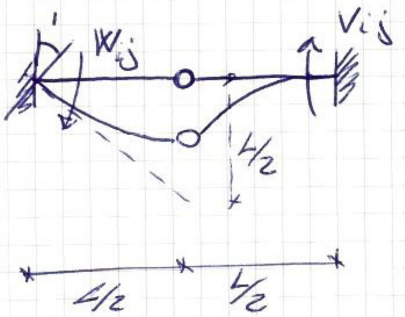
-sezione BC

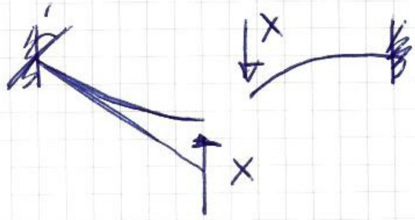
$b = 30\ cm$

$h = 50\ cm$

$\varepsilon_\varphi = L/(E_c \cdot b_r \cdot h_r^3) = 7{,}31 \cdot 10^{-12}$

L'asta AB si può considerare come unica determinando degli opportuni coeff. di rigidezza.





Eq. di congruenza nella cerniera $\frac{L}{2} - \frac{X\left(\frac{L}{2}\right)^3}{3EI} = \frac{X\left(\frac{L}{2}\right)^3}{3EI}$

$$\frac{\cancel{2}XL^3}{\cancel{24}_{12}EI} = \frac{L}{2}$$

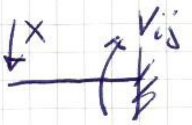
$$\Downarrow$$

$$X = \frac{\cancel{L}}{\cancel{2}} \cdot \frac{\cancel{12}^{6}EI}{L^{\cancel{3}2}} = \frac{6EI}{L^2}$$

$$W_{ij} - X\frac{L}{2} = 0$$

$$\Downarrow$$

$$W_{ij} = X\frac{L}{2} = \frac{\cancel{6}^{3}EI}{L^{\cancel{2}}} \cdot \frac{\cancel{L}}{\cancel{2}} = \frac{3EI}{L}$$



$$V_{ij} = X\frac{L}{2} = \frac{3EI}{L}$$

Data la simmetria dell'asta $W_{ij} = W_{ji}$

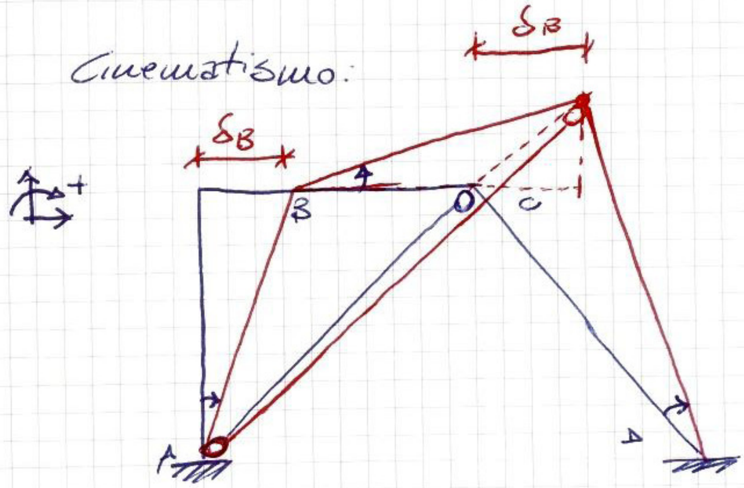
$V_{ij} = V_{ji}$

$$U_{ij} = \frac{W_{ij} + V_{ij}}{L} = \frac{6EI}{L^2}$$

- Calcolo del numero di nodi spostabili

$$3t - 2c = 9 - 8 = 1$$

Cinematismo:



Incognite:

$\{\varphi_B, \varphi_C, \delta_B\}$

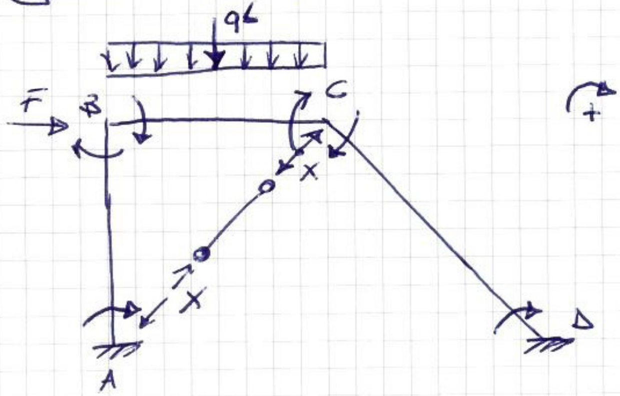
Definizione spostamenti virtuali e reali

	δ'_B
δ'_{AB}	1
δ'_{BC}	-1
δ'_{CD}	$\sqrt{2}$

	δ_B
δ_{AB}	1
δ_{BC}	-1
δ_{CD}	$\sqrt{2}$

Allungamento del pendolo $\Delta L = -\sqrt{2}\cdot\delta_B$

N.B. Si considerano gli allungamenti negativi.



Definizione delle equazioni risolutive:

$$\begin{cases} M_{BA} + M_{BC} = 0 \\ M_{CB} + M_{CD} = 0 \\ \dfrac{M_{AB}+M_{BA}}{L}\delta'_{AB} + \dfrac{M_{BC}+M_{CB}}{L}\delta'_{BC} + \dfrac{M_{CD}+M_{DC}}{\sqrt{2}L}\delta'_{CD} + F\delta'_B + \end{cases}$$

$$+ X\cdot\sqrt{2}\,\delta'_B - qL\cdot\frac{\delta'_B}{2} = 0$$

Passiamo alla risoluzione numerica:

- Calcolo coeff. di rigidezza

Asta AB : $L = 4000\ mm$ $E = 28500\ MPa$ $I = \frac{bh^3}{12} = 1{,}6\cdot 10^9\ mm^4$

$$W_{AB} = W_{BA} = \frac{3EI}{L} = 3{,}42\cdot 10^{10}\ Nmm$$

$$V_{AB} = V_{BA} = \frac{3EI}{L} = 3{,}42\cdot 10^{10}\ Nmm$$

$$U_{AB} = U_{BA} = \frac{6EI}{L^2} = 1{,}71 \cdot 10^7 \ N$$

ASTA BC : $L = 4000$ mm $\quad E = 28500$ MPa $\quad I = \frac{bh^3}{12} = 3{,}125 \cdot 10^9$ mm^4

$$W_{BC} = W_{CB} = \frac{4EI}{L} = 8{,}91 \cdot 10^{10} \ Nmm$$

$$V_{BC} = V_{CB} = \frac{2EI}{L} = 4{,}45 \cdot 10^{10} \ Nmm$$

$$U_{BC} = U_{CB} = \frac{6EI}{L^2} = 3{,}34 \cdot 10^7 \ N$$

$$\mu_{BC} = -\mu_{CB} = -\frac{qL^2}{12} = 2{,}67 \cdot 10^7 \ Nmm$$

ASTA CD: $L = \sqrt{2} L = 5656{,}85 \quad E = 28500$ MPa $\quad I = \frac{bh^3}{12} = 1{,}6 \cdot 10^9$ mm^4

$$\alpha_{DC}^* = \alpha_{DC} + \varepsilon\varphi = \frac{L}{3EI} + \varepsilon\varphi$$

$$\alpha_{DC}^* = 4{,}14 \cdot 10^{-11} + 7{,}31 \cdot 10^{-12} = 4{,}87 \cdot 10^{-11}$$

$$W_{CD} = \frac{\alpha_{DC}^*}{\alpha_{CD}\alpha_{DC}^* - \beta^2}$$

$$\alpha_{CD} = \frac{L}{3EI} = 4{,}14 \cdot 10^{-11}$$

$$\beta = \frac{L}{6EI} = 2{,}07 \cdot 10^{-11}$$

$$W_{CD} = \frac{4{,}87 \cdot 10^{-11}}{4{,}87 \cdot 10^{-11} \cdot 4{,}14 \cdot 10^{-11} - (2{,}07 \cdot 10^{-11})^2} = 3{,}07 \cdot 10^{10} \ Nmm$$

$$W_{DC} = \frac{4{,}14 \cdot 10^{-11}}{4{,}87 \cdot 10^{-11} \cdot 4{,}14 \cdot 10^{-11} - (2{,}07 \cdot 10^{-11})^2} = 2{,}61 \cdot 10^{10} \ Nmm$$

$$V_{CD} = V_{DC} = \frac{\beta}{\alpha_{CD}\alpha_{DC}^* - \beta^2} = \frac{2{,}07 \cdot 10^{-11}}{(\ldots\ldots\ldots)} = 1{,}30 \cdot 10^{10} \ Nmm$$

$$U_{CD} = \frac{W_{CD} + V_{CD}}{L} = 7{,}73 \cdot 10^6 \ N$$

$$U_{DC} = \frac{W_{DC} + V_{DC}}{L} = 6{,}91 \cdot 10^6\ N$$

Scrittura dei Momenti:

$$M_{AB} = V_{BA}\,\varphi_B - U_{BA}\,\delta_{AB} = 3{,}42 \cdot 10^{10}\,\varphi_B - 1{,}71 \cdot 10^7\,\delta_{AB}$$

$$M_{BA} = W_{BA}\,\varphi_B - U_{BA}\,\delta_{AB} = 3{,}42 \cdot 10^{10}\,\varphi_B - 1{,}71 \cdot 10^7\,\delta_{AB}$$

in termini di δ_B ricordando che $\delta_{AB} = \delta_B$

$$M_{AB} = 3{,}42 \cdot 10^{10}\,\varphi_B - 1{,}71 \cdot 10^7\,\delta_B$$

$$M_{BA} = 3{,}42 \cdot 10^{10}\,\varphi_B - 1{,}71 \cdot 10^7\,\delta_B$$

$$M_{BC} = W_{BC}\,\varphi_B + V_{CB}\,\varphi_C - U_{BC}\,\delta_{BC} + \mu_{BC} =$$
$$= 8{,}91 \cdot 10^{10}\,\varphi_B + 4{,}45 \cdot 10^{10}\,\varphi_C + 3{,}34 \cdot 10^7\,\delta_B - 2{,}67 \cdot 10^7$$

$$M_{CB} = W_{CB}\,\varphi_C + V_{BC}\,\varphi_B - U_{CB}\,\delta_{BC} + \mu_{CB} =$$
$$= 8{,}91 \cdot 10^{10}\,\varphi_C + 4{,}45 \cdot 10^{10}\,\varphi_B + 3{,}34 \cdot 10^7\,\delta_B + 2{,}67 \cdot 10^7$$

$$M_{CD} = W_{CD}\,\varphi_C - U_{CD}\,\delta_{CD} = 3{,}07 \cdot 10^{10}\,\varphi_C - 7{,}73 \cdot 10^6 \cdot \sqrt{2}\,\delta_B =$$
$$= 3{,}07 \cdot 10^{10}\,\varphi_C - 1{,}09 \cdot 10^7\,\delta_B$$

$$M_{DC} = V_{DC}\,\varphi_C - U_{DC}\,\delta_{CD} = 1{,}30 \cdot 10^{10}\,\varphi_C - 9{,}77 \cdot 10^6\,\delta_B$$

Scrittura del Sistema:

$$\begin{cases} 3{,}42 \cdot 10^{10}\,\varphi_B - 1{,}71 \cdot 10^7\,\delta_B + 8{,}91 \cdot 10^{10}\,\varphi_B + 4{,}45 \cdot 10^{10}\,\varphi_C + 3{,}34 \cdot 10^7\,\delta_B + \\ \quad - 2{,}67 \cdot 10^7 = 0 \\ 8{,}91 \cdot 10^{10}\,\varphi_C + 4{,}45 \cdot 10^{10}\,\varphi_B + 3{,}34 \cdot 10^7\,\delta_B + 2{,}67 \cdot 10^7 + 3{,}07 \cdot 10^{10}\,\varphi_C + \\ \quad - 1{,}09 \cdot 10^7\,\delta_B = 0 \\ \frac{M_{AB} + M_{BA}}{L}\,\cancel{\delta'_B} - \frac{M_{BC} + M_{CB}}{L}\,\cancel{\delta'_B} + \frac{M_{CD} + M_{DC}}{\cancel{\sqrt{2}}\,L} \cdot \cancel{\sqrt{2}}\,\cancel{\delta'_B} + F\,\cancel{\delta'_B} + \sqrt{2}\,X\,\cancel{\delta'_B} + \\ \quad \frac{qL}{2}\,\cancel{\delta'_B} = 0 \end{cases}$$

$$\begin{cases} 1{,}23\cdot10^{11}\varphi_B + 4{,}45\cdot10^{10}\varphi_C + 1{,}63\cdot10^{7}\delta_B - 2{,}67\cdot10^{7} = 0 \\ 4{,}45\cdot10^{10}\varphi_B + 1{,}20\cdot10^{11}\varphi_C + 2{,}25\cdot10^{7}\delta_B + 2{,}67\cdot10^{7} = 0 \\ \dfrac{3{,}42\cdot10^{10}\varphi_B - 1{,}71\cdot10^{7}\delta_B + 3{,}42\cdot10^{10}\varphi_B - 1{,}71\cdot10^{7}\delta_B}{4000} + \\ - \dfrac{8{,}91\cdot10^{10}\varphi_B + 4{,}45\cdot10^{10}\varphi_C + 3{,}34\cdot10^{7}\delta_B - 2{,}67\cdot10^{7} + 8{,}91\cdot10^{10}\varphi_C + 4{,}45\cdot10^{10}\varphi_B + 3{,}34\cdot10^{7}\delta_B + 2{,}67\cdot10^{7}}{4000} + \\ + \dfrac{3{,}07\cdot10^{10}\varphi_C - 1{,}09\cdot10^{7}\delta_B + 1{,}30\cdot10^{10}\varphi_C - 9{,}77\cdot10^{6}\delta_B}{4000} + 100000 + \\ + \sqrt{2}\left(\dfrac{E_p A_p\left(-\sqrt{2}\,\delta_B\right)}{L_p}\right) - 40000 = 0 \end{cases}$$

$$\begin{cases} 1{,}23\cdot10^{11}\varphi_B + 4{,}45\cdot10^{10}\varphi_C + 1{,}63\cdot10^{7}\delta_B - 2{,}67\cdot10^{7} = 0 \\ 4{,}45\cdot10^{10}\varphi_B + 1{,}20\cdot10^{11}\varphi_C + 2{,}25\cdot10^{7}\delta_B + 2{,}67\cdot10^{7} = 0 \\ 1{,}71\cdot10^{7}\varphi_B - 8550\,\delta_B - 3{,}34\cdot10^{7}\varphi_B - 3{,}34\cdot10^{7}\varphi_C - 16700\,\delta_B + \\ \quad + 1{,}09\cdot10^{7}\varphi_C - 5167{,}5\,\delta_B + 100000 - 5{,}07\cdot10^{4}\delta_B - 40000 = 0 \end{cases}$$

$$\begin{cases} 1{,}23\cdot10^{11}\varphi_B + 4{,}45\cdot10^{10}\varphi_C + 1{,}63\cdot10^{7}\delta_B = 2{,}67\cdot10^{7} \\ 4{,}45\cdot10^{10}\varphi_B + 1{,}20\cdot10^{11}\varphi_C + 2{,}25\cdot10^{7}\delta_B = -2{,}67\cdot10^{7} \\ -1{,}63\cdot10^{7}\varphi_B - 2{,}25\cdot10^{7}\varphi_C - 81067\,\delta_B = -60000 \end{cases}$$

Si risolve sotto forma matriciale:

$$\begin{bmatrix} 1{,}23\cdot10^{11} & 4{,}45\cdot10^{10} & 1{,}63\cdot10^{7} \\ 4{,}45\cdot10^{10} & 1{,}20\cdot10^{11} & 2{,}25\cdot10^{7} \\ -1{,}63\cdot10^{7} & -2{,}25\cdot10^{7} & -81067 \end{bmatrix} \begin{bmatrix} \varphi_B \\ \varphi_C \\ \delta_B \end{bmatrix} = \begin{bmatrix} 2{,}67\cdot10^{7} \\ -2{,}67\cdot10^{7} \\ -60000 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{cases} \varphi_B = 2{,}83\cdot10^{-4} \\ \varphi_C = -4{,}80\cdot10^{-4} \\ \delta_B = 0{,}8166\ \text{mm} \end{cases}$$

Calcolo Momenti: $\circlearrowright +$

$M_{AB} = -4{,}30$ kNm

$M_{BA} = -4{,}30$ kNm

$M_{BC} = 4{,}38$ kNm

(approssimazioni dei calcoli)

$M_{CB} = 23{,}75$ kNm

$M_{CD} = -23{,}65$ kNm

(approssimazioni dei calcoli)

$M_{DC} = -14{,}22$ kNm

Tagli: $\uparrow\overset{+}{\square}\downarrow$

$$T_{AB} = T_{BA} = -\frac{M_{AB}+M_{BA}}{L} = 2{,}14\ \text{kN}$$

$$T_{BC} = \frac{qL}{2} - \frac{M_{BC}+M_{CB}}{L} = 32{,}97\ \text{kN}$$

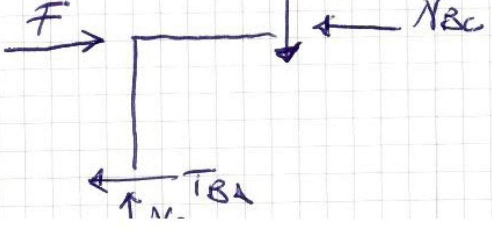
$$T_{CB} = -\frac{qL}{2} - \frac{M_{BC}+M_{CB}}{L} = -47{,}03\ \text{kN}$$

$$T_{CD} = T_{DC} = -\frac{M_{CD}+M_{DC}}{\sqrt{2}L} = 6{,}69\ \text{kN}$$

Calcolo Sforzi Normali $\rightarrow\overset{+}{\square}\leftarrow$

$$X = \frac{E_p A_p}{L_p}\cdot\Delta L_p = \frac{E_p A_p}{L_p}\left(-\sqrt{2}\,\delta_B\right) = -29{,}30\ \text{kN}$$

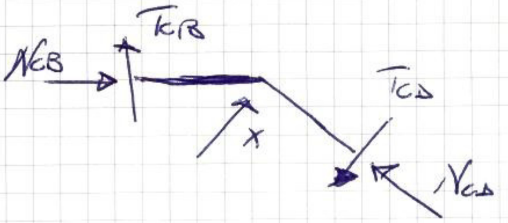
Nodo B



$$N_{BC} = F - T_{BA} = 97{,}86\ \text{kN}$$

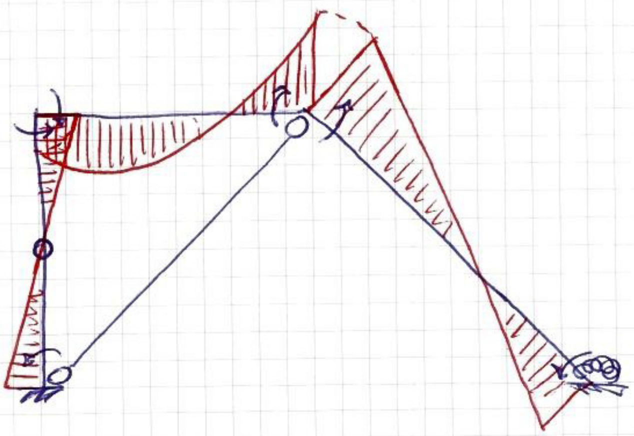
$$N_{BA} = T_{BC} = 32{,}97\ \text{kN}$$

Nodo C

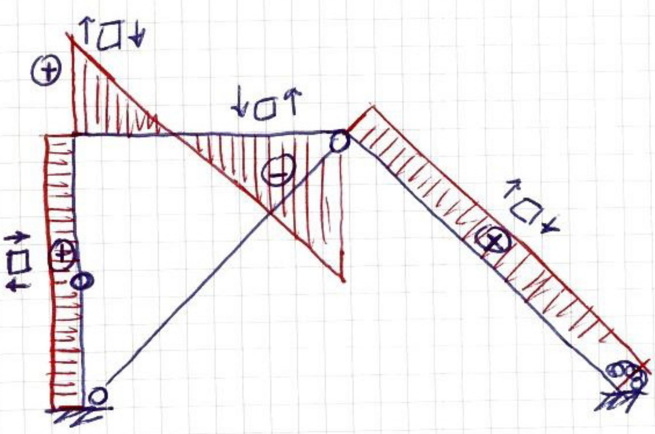


$$\begin{cases} \dfrac{N_{CD}}{\sqrt{2}} + \dfrac{X}{\sqrt{2}} - \dfrac{T_{CD}}{\sqrt{2}} + T_{CB} = 0 \\ N_{CB} + \dfrac{X}{\sqrt{2}} - \dfrac{T_{CD}}{\sqrt{2}} - \dfrac{N_{CD}}{\sqrt{2}} = 0 \end{cases}$$

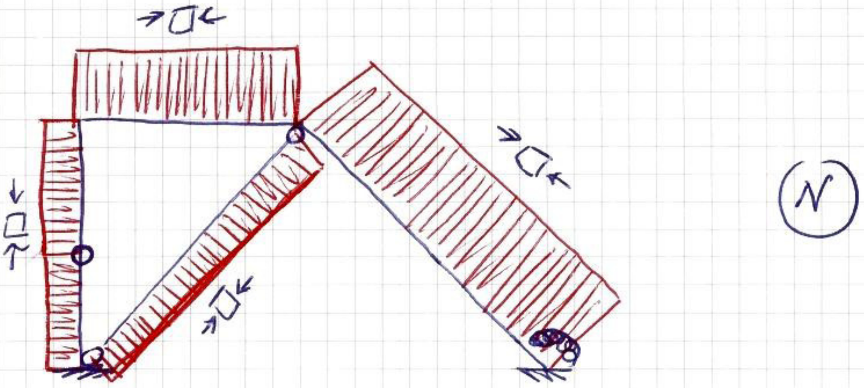
$$\begin{cases} N_{CD} = \left[\dfrac{T_{CD}}{\sqrt{2}} - T_{CB} - \dfrac{X}{\sqrt{2}}\right]\sqrt{2} = 102{,}50\ KN \\ N_{CB} = \dfrac{T_{CD}}{\sqrt{2}} - \dfrac{X}{\sqrt{2}} + \dfrac{N_{CD}}{\sqrt{2}} = 97{,}93\ KN \cong N_{BC} \quad OK! \end{cases}$$



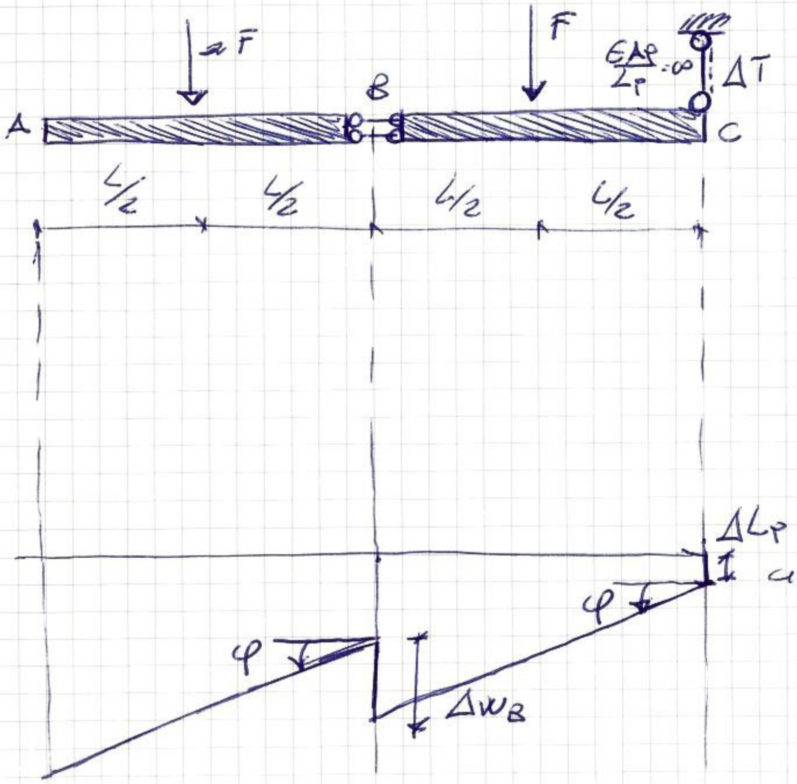
M



T


N

Esercizio n. 3 del 22/04/2008



$K_o = 0{,}01\ N/mm^3$

$B = 100\ cm$

$L_p = 2\ m$

$L = 5\ m$

$F = 100\ KN$

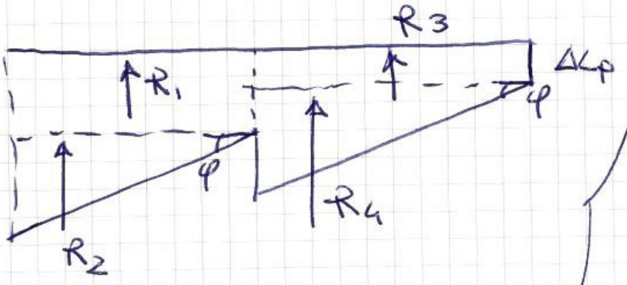
$\Delta T = 20°C \quad \alpha = 0{,}00001\ °C^{-1}$

ΔL_p é noto data l'inestensibilitá del pendolo

$$\Delta L_p = \alpha \Delta T L_p = 0{,}00001 \cdot 20 \cdot 2000 = 0{,}4\ mm$$

incognite $\{\varphi, \Delta w_B\}$

Consideriamo le reazioni del terreno:



$$\begin{cases} R_3 = \Delta L_p \cdot L \cdot K_o \cdot B \\ R_4 = \varphi \cdot L \cdot \frac{L}{2} \cdot K_o \cdot B \\ R_1 = (\varphi L + \Delta L_p - \Delta w_B) L K_o B \\ R_2 = \varphi L \frac{L}{2} \cdot K_o \cdot B \end{cases}$$

$$\begin{cases} R_3 = 0{,}4 \cdot 5000 \cdot 0{,}01 \cdot 1000 = 20\,000\ [N] \\ R_4 = \varphi \cdot \frac{5000^2}{2} \cdot 0{,}01 \cdot 1000 = \varphi \cdot 1{,}25 \cdot 10^8 \\ R_1 = \left[\varphi \cdot 5000 + 0{,}4 - \Delta w_B\right] \cdot 5000 \cdot 0{,}01 \cdot 1000 = 2{,}5 \cdot 10^8 \varphi + 2 \cdot 10^4 - 5 \cdot 10^4 \Delta w_B \\ R_2 = \varphi \cdot \frac{5000^2}{2} \cdot 0{,}01 \cdot 1000 = \varphi \cdot 1{,}25 \cdot 10^8 \end{cases}$$

Equazioni Risolutive

- Equ. alla traslazione locale nel tronco $(\downarrow +)$

$$2F - R_1 - R_2 = 0$$

- Equ. alla rotazione globale $(\curvearrowright +)$

$$2F \cdot \frac{3}{2}L + F \cdot \frac{L}{2} - R_1 \cdot \frac{3}{2}L - R_3 \cdot \frac{L}{2} - R_2 \cdot \frac{5}{3}L - R_4 \cdot \frac{2}{3}L = 0$$

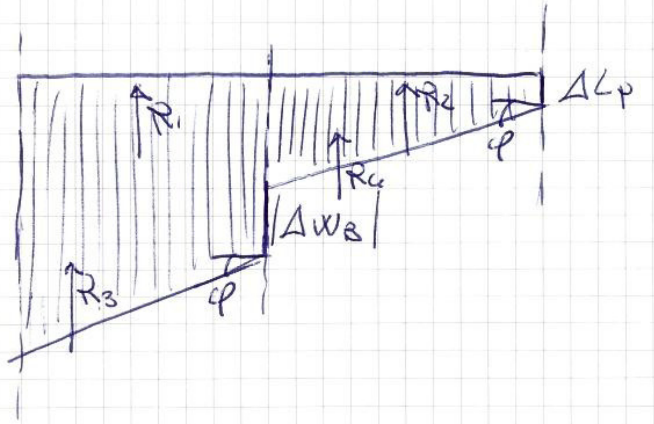
$$\begin{cases} 2 \cdot 10^5 - 2{,}5 \cdot 10^8 \varphi - 2 \cdot 10^4 + 5 \cdot 10^4 \Delta w_B - 1{,}25 \cdot 10^8 \varphi = 0 \\ 1{,}5 \cdot 10^9 + 2{,}5 \cdot 10^8 - (2{,}5 \cdot 10^8 \varphi + 2 \cdot 10^4 - 5 \cdot 10^4 \Delta w_B) \cdot 7{,}5 \cdot 10^3 - 2 \cdot 10^4 \cdot 2{,}5 \cdot 10^3 + \\ \qquad - 1{,}25 \cdot 10^8 \varphi \cdot 8{,}33 \cdot 10^3 - 1{,}25 \cdot 10^8 \cdot \varphi \cdot 3{,}33 \cdot 10^3 = 0 \end{cases}$$

$$\begin{cases} -3{,}5 \cdot 10^8 \varphi + 5 \cdot 10^4 \Delta w_B = -1{,}8 \cdot 10^5 \\ -3{,}33 \cdot 10^{12} \varphi - 3{,}75 \cdot 10^9 \Delta w_B = -1{,}55 \cdot 10^9 \end{cases}$$

$$\begin{bmatrix} -3{,}5 \cdot 10^8 & 5 \cdot 10^4 \\ -3{,}33 \cdot 10^{12} & -3{,}75 \cdot 10^9 \end{bmatrix} \begin{bmatrix} \varphi \\ \Delta w_B \end{bmatrix} = \begin{bmatrix} -1{,}8 \cdot 10^5 \\ -1{,}55 \cdot 10^9 \end{bmatrix}$$

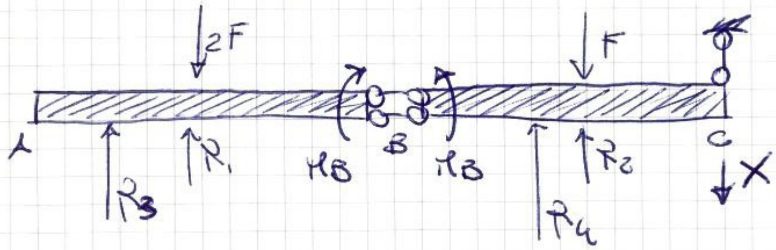
$$\begin{cases} \varphi = 5{,}09 \cdot 10^{-4} \\ \Delta w_B = -0{,}038 \text{ mm} \end{cases}$$

Cinematismo reale



$R_1 = 149{,}15$ kN
$R_2 = 63{,}63$ kN
$R_3 = 20$ kN
$R_4 = 63{,}63$ kN

Restano da determinare la reazione del pendolo esterno e il momento del pendolo interno



Eq. locale alla rotazione 2° tratto (↻+)

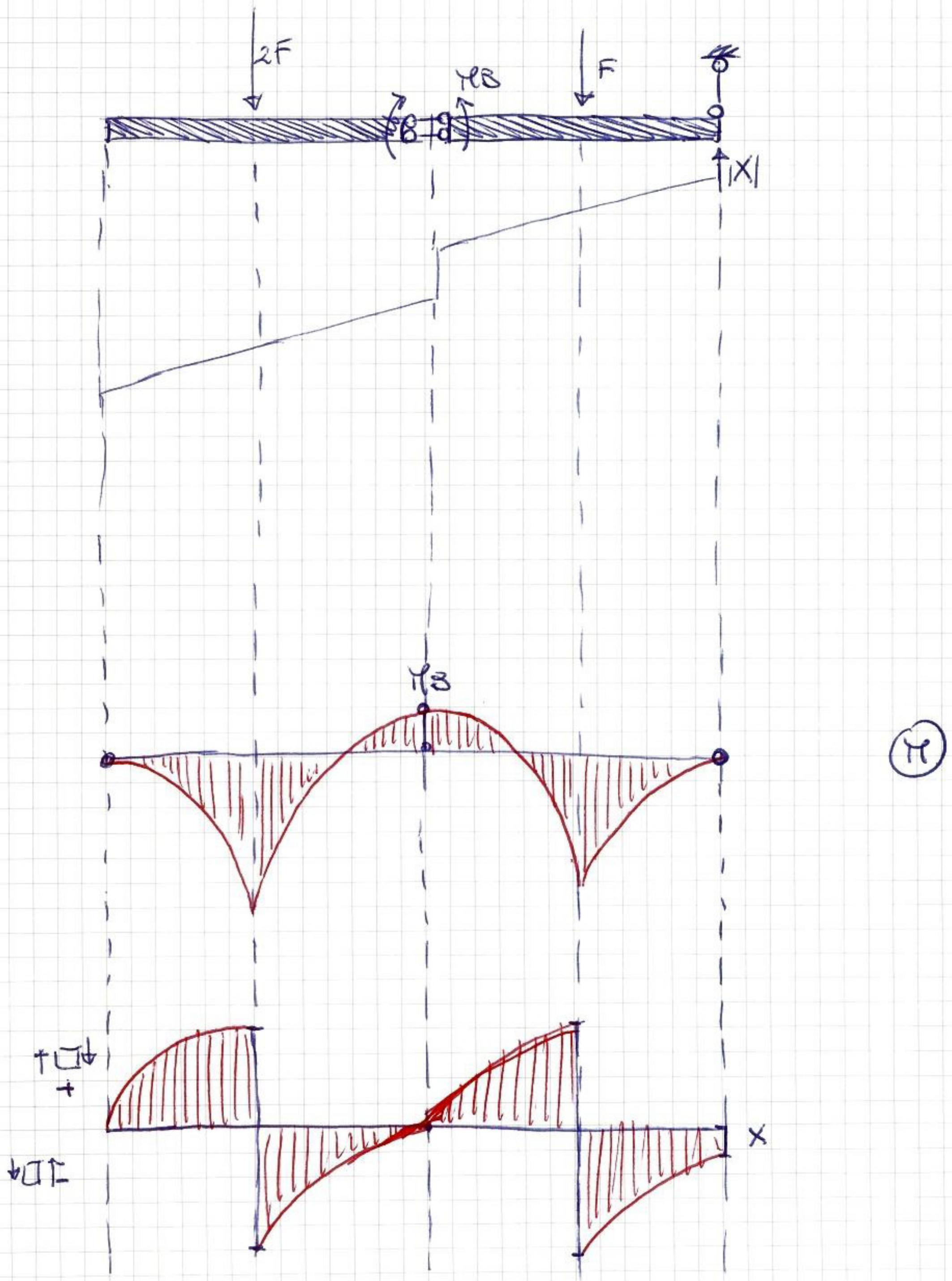
$$R_1 \cdot \frac{L}{2} + R_3 \frac{2}{3}L - 2F\frac{L}{2} + M_B = 0$$

$$M_B = 2F\frac{L}{2} - R_1\frac{L}{2} - R_3\frac{2}{3}L = 60{,}46 \text{ kNm}$$

Eq. globale alla traslazione (↓+)

$$2F + F + X - R_1 - R_3 - R_2 - R_4 = 0$$

$$X = R_1 + R_3 + R_2 + R_4 - 3F = -3{,}59 \text{ kN}$$ => X ha verso opposto


2F
F
MB
B
B
|X|
MB
M
+
X