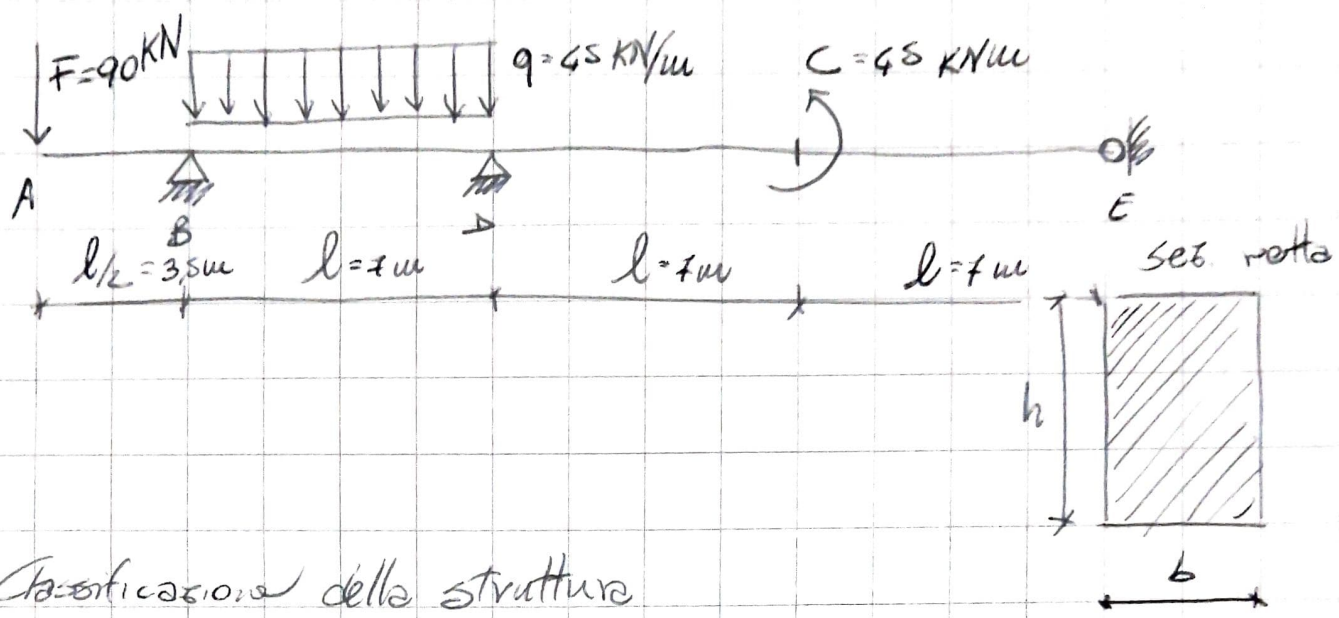
Esercizio - Trave continua e verifica di sicurezza



① Classificazione della struttura

$l - i = 3t - s$ (problema piano - 3 g.d.l. (estensionale e flessionale))

$3t = 3$

$s = 6$ (sono 3 cerniere con molteplicità 2)

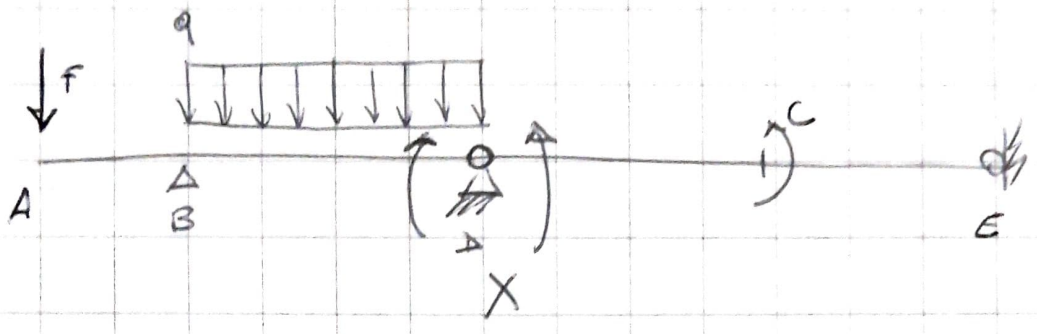
$l = 0 \Rightarrow 0 - i = 3 - 6 \Rightarrow i = 3$

Considerando che non vi sono forze esterne di tipo estensionale si possono considerare solo 2 g.d.l. flessionali (abbassamenti e rotazioni) e il vincolo delle cerniere solo in direzione ortogonale alla trave (il risultato del problema estensionale è nullo)

$l - i = 2t - s \Rightarrow 0 - i = 2 - 3 \Rightarrow i = 1$

(2) Valutazione delle reazioni vincolari

Struttura 1 volta iperstatica → creare 1 sconnessione e inserire l'incognita iperstatica duale



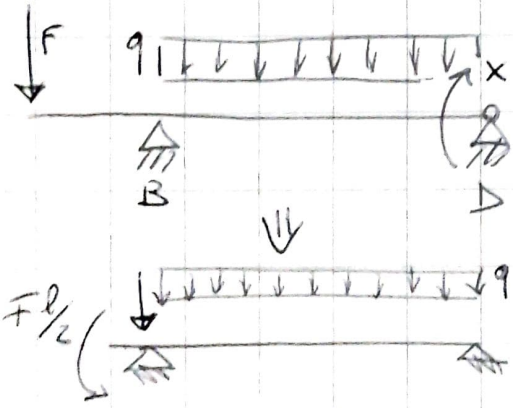
Si va a ristabilire la congruenza delle rotazioni in D (a sinistra e destra) per valutare l'incognita iperstatica

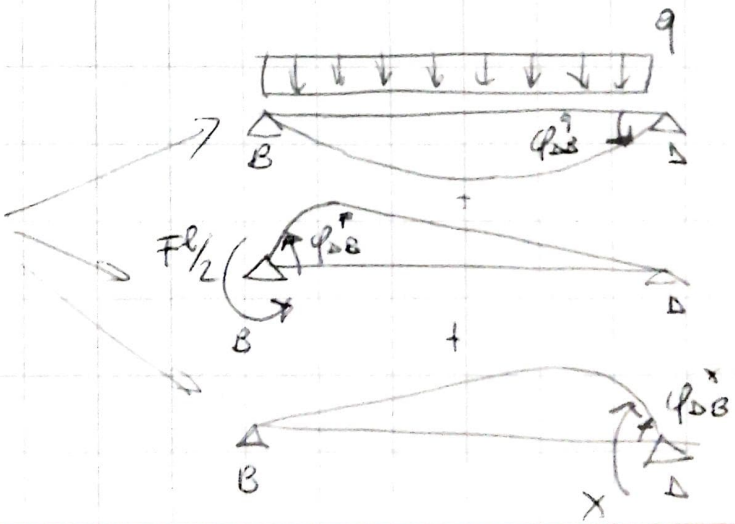
$$\varphi_{DB} = \varphi_{DE}$$

↓ sovrapposizione degli effetti

$$\varphi_{DB} = \varphi_{DB}^{(X)} + \varphi_{DB}^{(q)} + \varphi_{DB}^{(F)}$$

$$\varphi_{DE} = \varphi_{DE}^{(X)} + \varphi_{DE}^{(C)}$$





$$\varphi_{DB} = -\frac{Xl}{3EI} + \frac{ql^3}{24EI} - \frac{Fl}{2}\frac{l}{6EI}$$

rot. ↺ positive

$$\varphi_{DC} = \frac{X \cdot 2\ell}{3EI} - \frac{C(2\ell)}{24EI} \qquad (12)$$

$$\varphi_{DB} = \varphi_{DC}$$

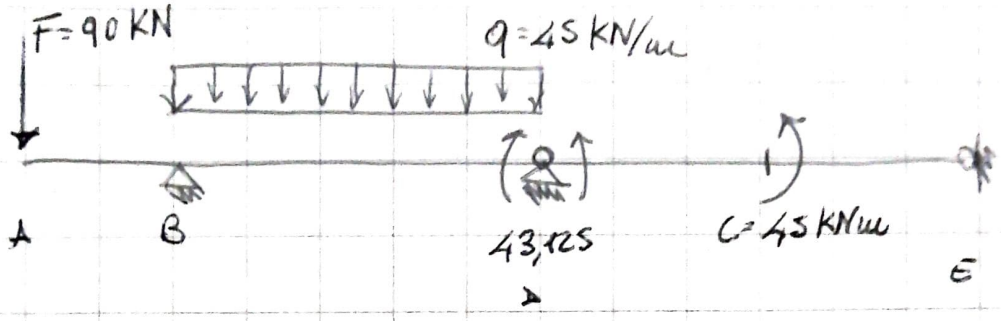
$$-\frac{X\ell}{3EI} + \frac{q\ell^2}{24EI} - \frac{F\ell^2}{12EI} = \frac{2X\ell}{3EI} - \frac{C\ell}{12EI} \qquad (8,\ 4,\ 4)$$

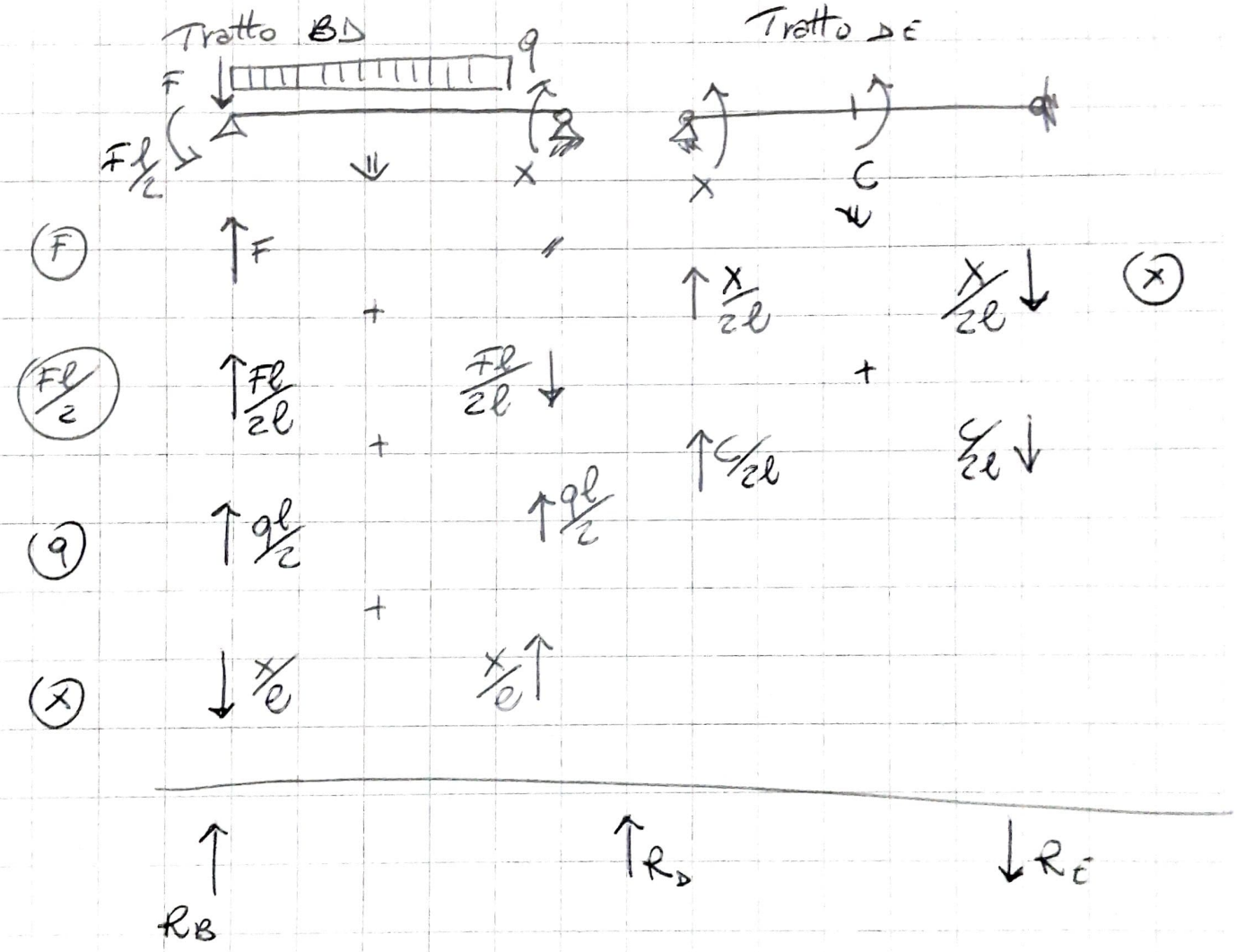
$$-X + \frac{q\ell^2}{8} - \frac{F\ell}{4} = 2X - \frac{C}{4}$$

$$3X = \frac{q\ell^2}{8} - \frac{F\ell}{4} + \frac{C}{4}$$

$$X = \frac{1}{3}\left[\frac{45 \cdot 7^2}{8} - \frac{90 \cdot 7}{4} + \frac{48}{4}\right] = \frac{2205 - 1206 + 90}{24} = \frac{345}{8} = 43{,}125\ kN$$

Procediamo al calcolo delle reazioni vincolari sempre con la sovrapposizione degli effetti.

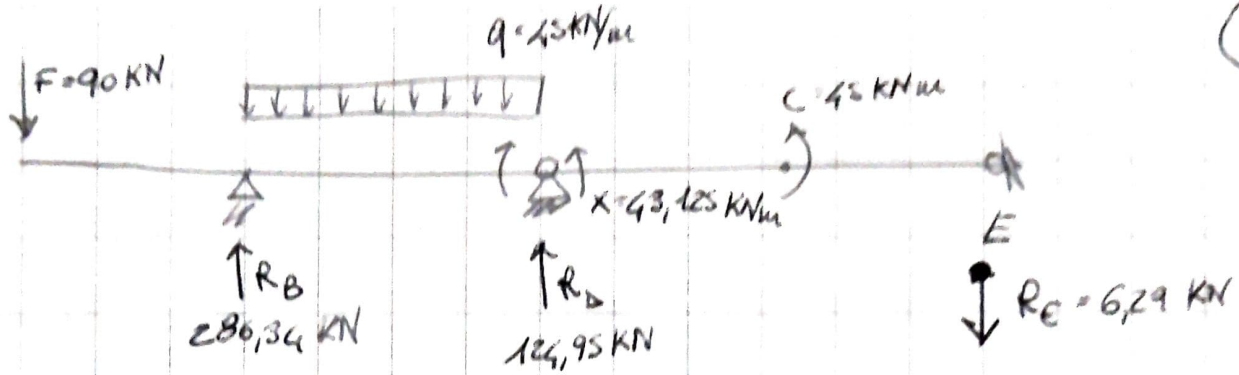




$$R_B = F + \frac{F}{2} + \frac{9l}{2} - \frac{X}{l} = 90 + 45 + 157{,}5 - 6{,}16 = 286{,}34 \text{ kN}$$

$$R_D = -\frac{F}{2} + \frac{9l}{2} + \frac{X}{l} + \frac{X}{2l} + \frac{C}{2l} = 124{,}95 \text{ kN}$$

$$R_E = \frac{X}{2l} + \frac{C}{2l} = 6{,}29 \text{ kN}$$



③ Diagrammi delle sollecitazioni N, T, M

T

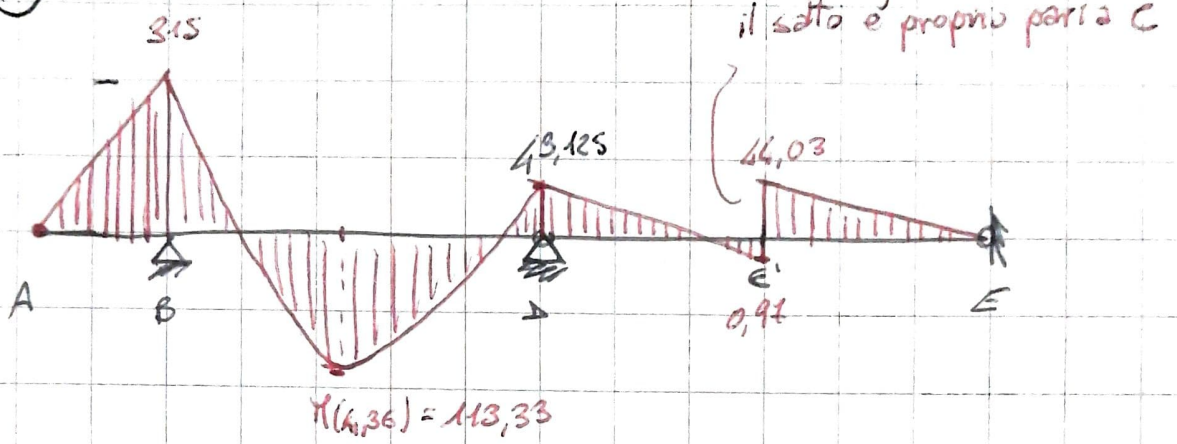
$T_A = -F = -90\ KN$

$T_B^S = T_A = -90\ KN$ $\quad T_B^A = T_B^S + R_B = 196,34\ KN$

$T_D^S = T_B^A - q \cdot \ell = -118,66\ KN$ $\quad T_D^A = T_D^S + R_D = 6,29\ KN$

$T_C = R_E = 6,29\ KN$

(4)



$M_A = 0$ $\qquad M_E = 0$

$M_B^S = F \cdot \ell/2 = 315$ kNm ↺

$M_B^D = M_B^S = 315$ kNm

$M_D^S = F(\ell/2 + \ell) - R_B \cdot \ell + q\ell^2/2 = 43{,}125$ kNm ↺ $= X$

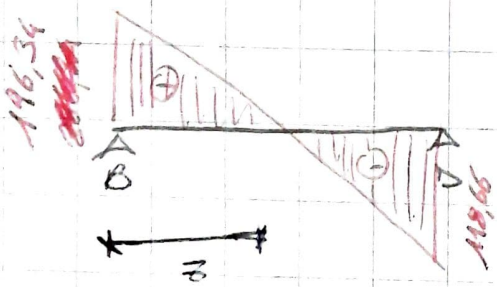
← procediamo da destra

$M_C^D = R_E \cdot \ell = 44{,}03$ ↻

$M_C^S = R_E \cdot \ell - C = -0{,}97$ kN ↻

Calcolo z di M_{max} in campata BD

Guardando il taglio

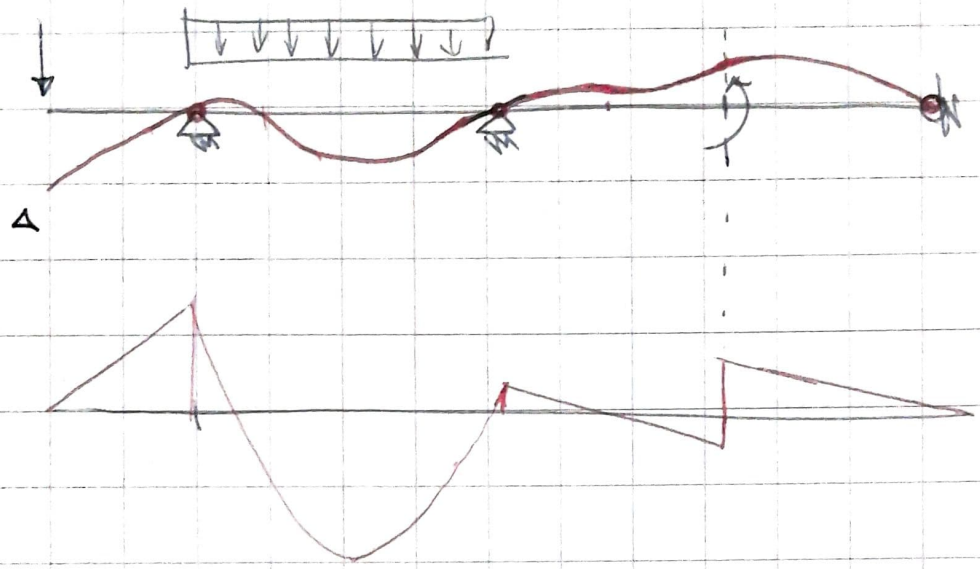


~~$\frac{196{,}36}{z}$~~ $= \frac{196{,}39 + 118{,}66}{7\,m}$

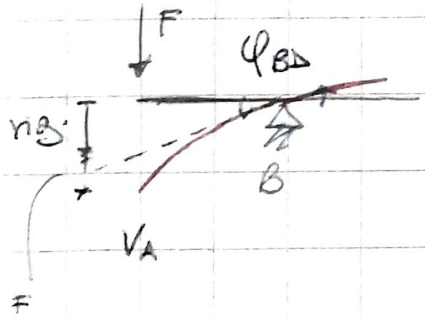
$z = 4{,}36$ m

$M(z) = F\left(\frac{\ell}{2} + 4{,}36\right) - R_B \cdot 4{,}36 + \frac{9 \cdot 4{,}36^2}{2} = -113{,}33$ ↑

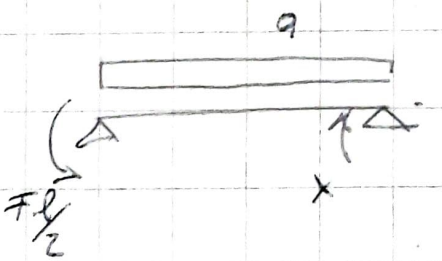
④ Disegno della deformata a mio piacere



⑤ Valutazione dello spostamento in A



somma di una rotazione rigida dell'estremo B e dell'abbassamento dell'estremità della mensola dovuta a F

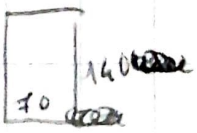


$$\varphi_{BD} = \varphi_{BD}^{(F)} + \varphi_{BD}^{(q)} + \varphi_{BD}^{(x)}$$

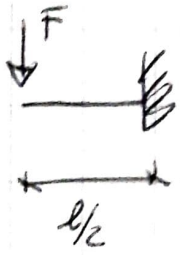
$$\varphi_{BD} = \frac{F\frac{l}{2}}{2}\frac{l}{3EI} - \frac{q l^3}{24EI} + \frac{X l}{6EI} = 0{,}0423 \text{ rad} = 2{,}4236^\circ$$

$E = 210\,000\ N/mm^2$

$I = \frac{b h^3}{12} = \frac{40 \cdot 140^3}{12} = 16\,006666{,}67\ mm^4$



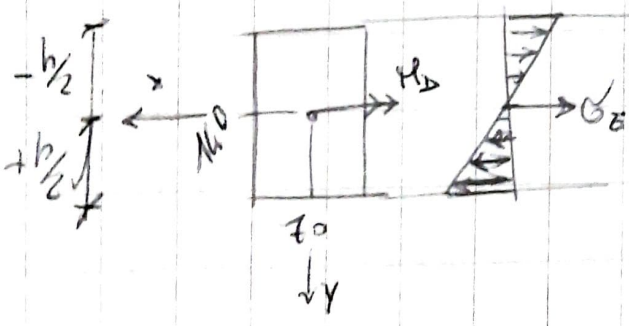
$$V_A^{(F)} = \frac{F\ell^3}{3EI} \Rightarrow \frac{F(\ell/2)^3}{3EI}$$



$$V_A^{(F)} = 382{,}65 \text{ mm}$$

$$V_A^{(Tot)} = V_A^{(F)} + \varphi_{BD} \cdot \ell/2 = 382{,}65 + 0{,}0423 \cdot 3500 = 530{,}7 \text{ mm}$$

⑥ Tensione normale massima e minima nella sezione Δ



$M_\Delta = X = -43{,}125 \text{ kNm}$

$$\sigma_z = \frac{N}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

Termine flessione fuori dal piano del disegno

$N_z = 0$

$$\sigma_z = \frac{M_x}{I_x} y$$

$$\sigma_z(z = -h/2) = -\frac{43{,}125}{1{,}6\,10^{-5}}\left(\frac{140}{-2}\right) = 188{,}67 \text{ N/mm}^2$$

$$\sigma_z(z = h/2) = -\frac{43{,}125}{1{,}6\,10^{-5}}\left(\frac{140}{2}\right) = 188{,}67 \text{ N/mm}^2$$

(Fe360)

Verifica di resistenza $\sigma_{eq} = \sigma_z = 188{,}67 > \sigma_{adm} = \frac{235}{1{,}5} = 160 \text{ N/mm}^2$ ← coeff. sic.

Non verificata