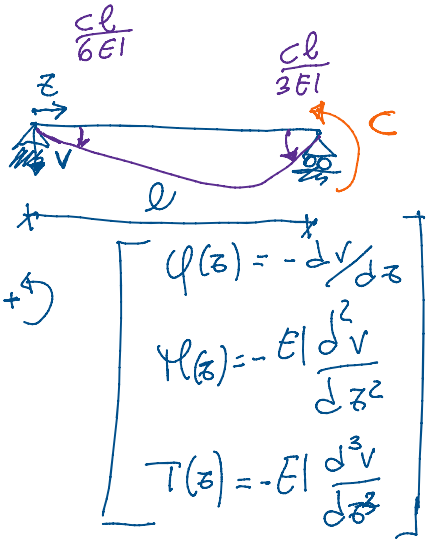
LEZIONE DEL 04.06.2020

giovedì 4 giugno 2020 14:18

Linea elastica

$$\frac{d^4 v}{dz^4} = -\frac{q(z)}{EI}$$

$$v(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \frac{q z^4}{24 EI}$$



$$\varphi(z) = -dv/dz$$

$$M(z) = -EI\frac{d^2 v}{dz^2}$$

$$T(z) = -EI\frac{d^3 v}{dz^3}$$

$z=0$

$$\begin{cases} v(z=0) = 0 \Rightarrow a_0 = 0 \\ M(z=0) = 0 \Rightarrow a_2 + a_3 z = 0 \Rightarrow a_2 = 0 \end{cases}$$

$z=l$

$$\begin{cases} v(z=l) = 0 \Rightarrow a_1 l + a_3 l^3 = 0 \\ M(z=l) = C \Rightarrow -EI\,6\,a_3 l = C \end{cases}$$

$$a_3 = -\frac{C}{6EI\,l} \qquad a_1 = \frac{Cl}{6EI}$$

$$v(z) = \frac{Cl}{6EI} z - \frac{C}{6EI\,l} \cdot z^3$$

$$\varphi(z) = -\frac{dv}{dz} = -\frac{Cl}{6EI}\cdot\left[1 - 3\left(\frac{z}{l}\right)^2\right]$$

$$-\frac{Cl}{6EI} \qquad [1-3] = +2$$

$$\varphi(0) = -\frac{Cl}{6EI}$$

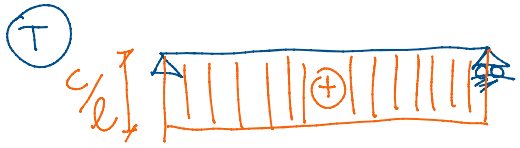
$$\varphi(l) = +\frac{Cl}{3EI}$$

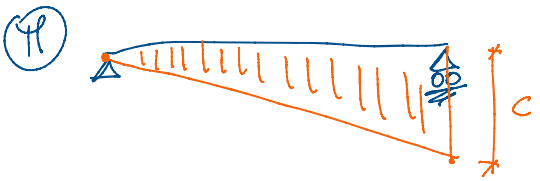
$$M(z) = -EI\frac{d^2 v}{dz^2} = -\frac{C}{6}\left[-6\frac{z}{l}\right] = \frac{C z}{l}$$

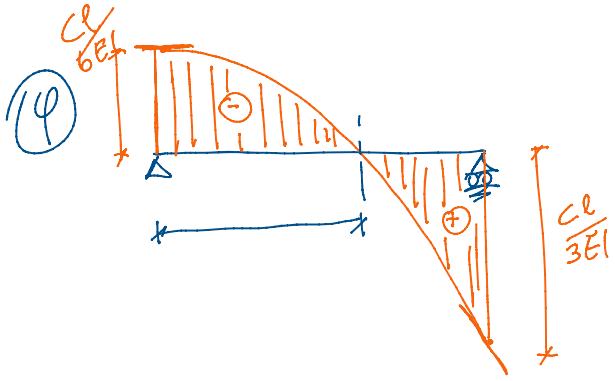
$$M(0) = 0$$

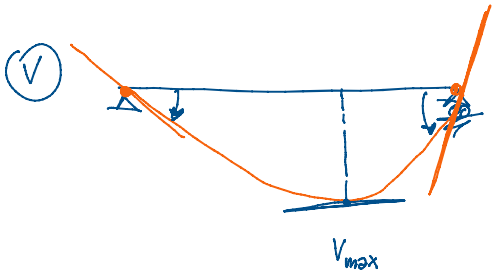
$$M(l) = C$$

$$T(z) = -EI\frac{d^3 v}{dz^3} = \frac{C}{l}$$





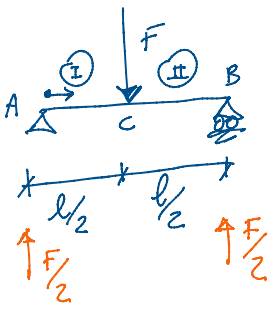




$$\varphi = -\frac{C\ell}{6EI}\left[1 - 3\left(\frac{z}{\ell}\right)^2\right] = 0 \quad \Rightarrow \quad z = \frac{\sqrt{3}}{3}\ell$$

$$V_{max} = V\left(z = \frac{\sqrt{3}}{3}\ell\right) = \frac{C\ell}{6EI}\cdot\frac{\sqrt{3}}{3}\ell - \frac{C}{6EI\cdot\ell}\left(\frac{\sqrt{3}}{3}\ell\right)^3 =$$

$$V_{max} = \frac{C\ell}{6EI}\frac{\sqrt{3}}{3}\ell - \frac{C}{6EI}\ell^2\left(\frac{\sqrt{3}}{3}\right)^3 \rightarrow$$



$$\frac{d^4V}{dz^4} = -\frac{q(z)}{EI} = 0$$

Tratto AC $\left(0 \le z \le \frac{\ell}{2}\right)$

- $V_1(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3$

Tratto CB $\left(\frac{\ell}{2} \le z \le \ell\right)$

- $V_2(z) = b_0 + b_1 z + b_2 z^2 + b_3 z^3$

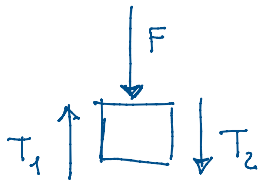
8 costanti incognite

$z = 0$ $\begin{cases} V_1(0) = 0 \\ M_1(0) = 0 \end{cases}$

$z = \ell$ $\begin{cases} V_2(\ell) = 0 \\ M_2(\ell) = 0 \end{cases}$

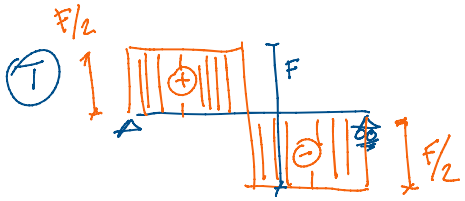
$z = \frac{\ell}{2}$ $\begin{cases} V_1\left(\frac{\ell}{2}\right) = V_2\left(\frac{\ell}{2}\right) \\ \varphi_1\left(\frac{\ell}{2}\right) = \varphi_2\left(\frac{\ell}{2}\right) \\ M_1\left(\frac{\ell}{2}\right) = M_2\left(\frac{\ell}{2}\right) \\ T_2\left(\frac{\ell}{2}\right) - T_1\left(\frac{\ell}{2}\right) + F = 0 \end{cases}$

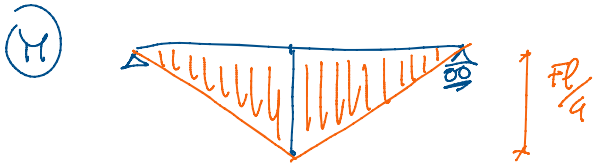
$\begin{cases} M_2(l) = 0 \\ T_2\left(\frac{l}{2}\right) - T_1\left(\frac{l}{2}\right) + F = 0 \end{cases}$

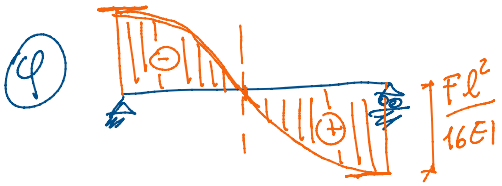


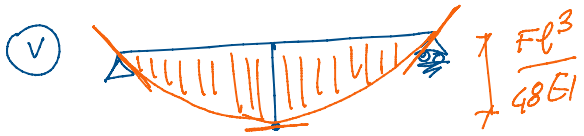
$a_0 = 0$ $\quad a_1 = \frac{Fl^2}{16EI}$ $\quad b_1 = \frac{3Fl^2}{16EI}$ $\quad b_3 = \frac{F}{12EI}$

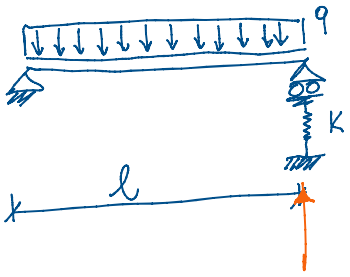
$a_2 = 0$ $\quad a_3 = -\frac{F}{12EI}$ $\quad b_2 = -\frac{Fl}{4EI}$ $\quad b_0 = -\frac{1}{48}$











$$v(l) = -\frac{R}{K}$$

$$v(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \frac{q z^4}{24EI}$$

$z = 0 \begin{cases} v(0) = 0 \\ M(0) = 0 \end{cases}$ $\qquad z = l \begin{cases} v(l) = -\frac{R}{K} = -\frac{T(l)}{K} \\ M(l) = 0 \end{cases}$

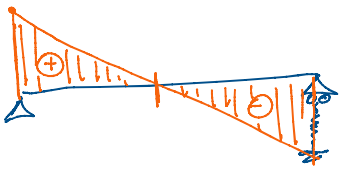
$$\left.\begin{aligned} \varphi(z) &= -a_1 - 3a_3 z^2 - \frac{q z^3}{6EI} \\ M(z) &= EI\left(-6a_3 z - \frac{q z^2}{2EI}\right) \end{aligned}\right\} \rightarrow\rightarrow \begin{aligned} v(z) &= \left(\frac{q}{2K} + \frac{ql^3}{24EI}\right) z - \frac{ql}{12EI} z^3 + \frac{q z^4}{24EI} \\ \varphi(z) &= -\left(\frac{q}{2K} + \frac{ql^3}{24EI}\right) + \frac{ql}{4EI} z^2 - \frac{q z^3}{6EI} \end{aligned}$$

$$\left.\begin{aligned} M(z) &= EI\left(-6a_3 z - \frac{q_0}{2EI}\right) \\ T(z) &= EI\left(-6a_3 - \frac{qz}{EI}\right) \end{aligned}\right\} \Rightarrow \varphi(z) = -\left(\frac{q}{2k} + \frac{ql^2}{24EI}\right) + \frac{q_0}{4EI} z^2 - \frac{q}{6EI} z^3$$

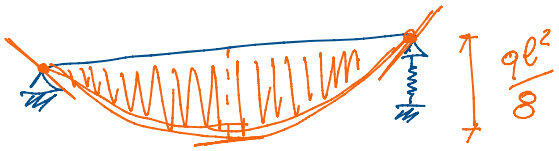
$$M(z) = EI\left(\frac{ql}{2EI} z - \frac{qz^2}{2EI}\right) = \frac{qlz}{2} - \frac{qz^2}{2}$$

$$T(z) = \frac{ql}{2} - qz$$

(T)

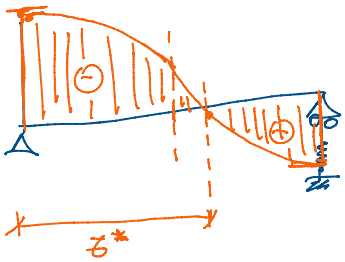


(M)



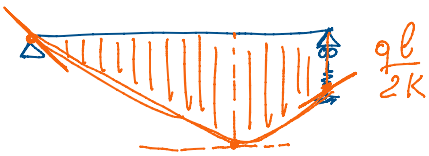
$-\frac{q}{2k} - \frac{ql^3}{24EI}$

(φ)



$$\varphi = -\frac{q}{2k} + \frac{ql^3}{24EI}$$

(V)



$$V(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3$$

$z = 0$ $\begin{cases} V(0) = 0 \Rightarrow a_0 = 0 \\ \varphi(0) = 0 \Rightarrow a_1 = 0 \end{cases}$ $z = l$ $\begin{cases} V(l) = V_B^* \\ M(l) = 0 \end{cases}$

$$\Rightarrow a_3 = -\frac{V_B^*}{2l^3}$$

$$\Rightarrow a_2 = -3a_3 l \Rightarrow a_2 = \frac{3}{2}\frac{V_B^*}{l^2}$$

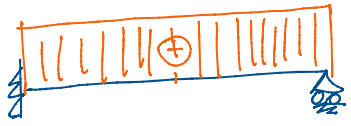
$$V(z) = \frac{3}{2}\frac{V_B^*}{l^2} z^2 - \frac{V_B^*}{2l^3} z^3$$

$$\varphi(z) = -3\frac{V_B^*}{2l}\left[l\frac{z}{l} - \left(\frac{z}{l}\right)^2\right]$$

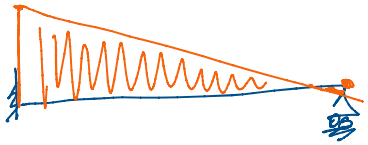
$$M(z) = -EI\,\frac{3V_B^*}{l^2}\left[1 - \frac{z}{l}\right]$$

$$T(z) = EI\,\frac{3V_B^*}{l^3}$$

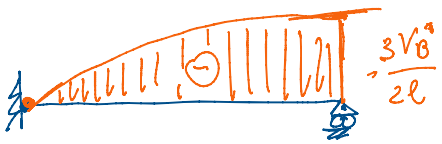
(T)



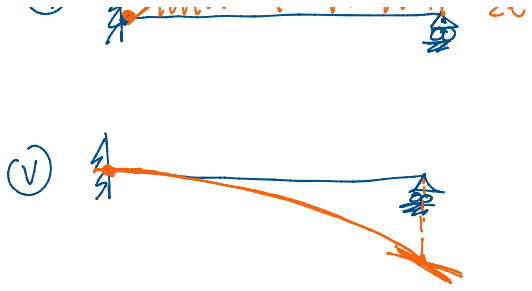
(M)



(φ)

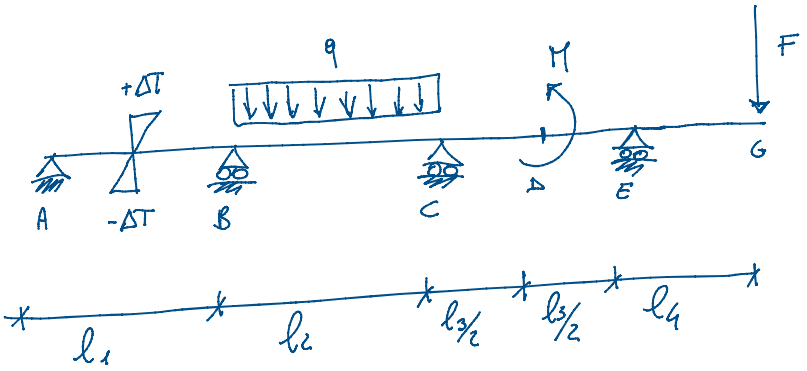


$$T(z) = EI\,\frac{3v_B^*}{l^3}$$



La lezione riprenderà alle ore 16:10

Trave continua



$$\theta^* = -\frac{2\alpha\Delta t}{h}$$

$l_1 = 4\ m$ $\quad q = 20\ kN/m$

$l_2 = 6\ m$ $\quad M = 10\ kN\,m$

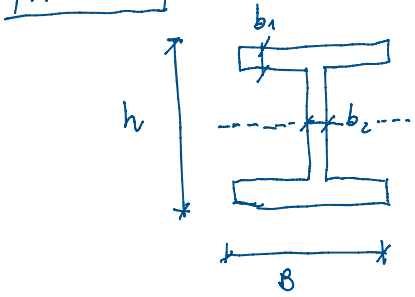
$l_3 = 4\ m$ $\quad F = 25\ kN$

$l_4 = 2\ m$ $\quad \Delta t = 15\ °C$

$E = 2{,}1 \cdot 10^5\ N/mm^2$ $\quad \alpha = 1{,}2 \cdot 10^{-5}\ °C$

$\sigma_{amm} = 160\ N/mm^2$

IPE 300



$h = 300\ mm$

$B = 150\ mm$

$b_1 = 10{,}7\ mm$

$b_2 = 7{,}1\ mm$

$A = 51{,}88\ cm^2$

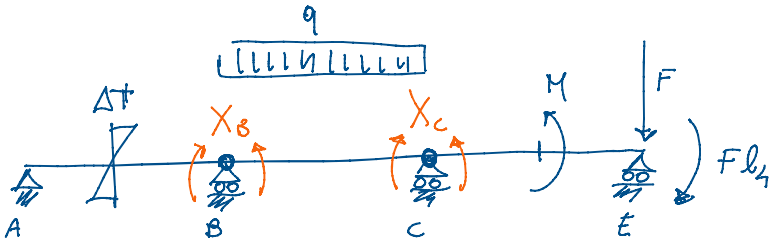
$I_x = 7999\ cm^4$

$W_x = \frac{I_x}{h/2} = 533{,}27\ cm^3$

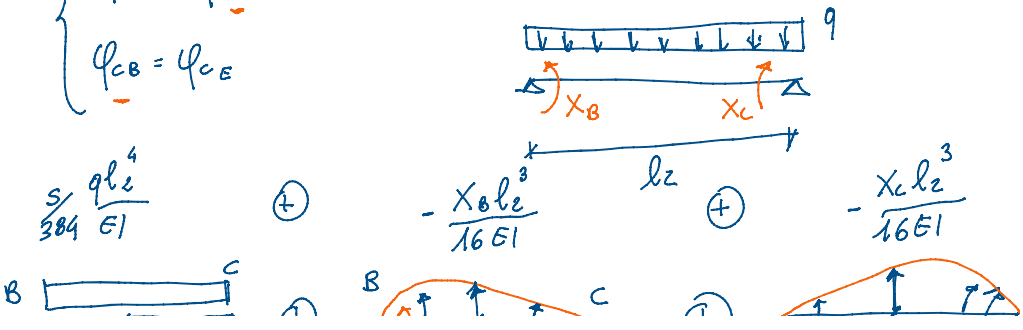
① Determinare le incognite iperstatiche

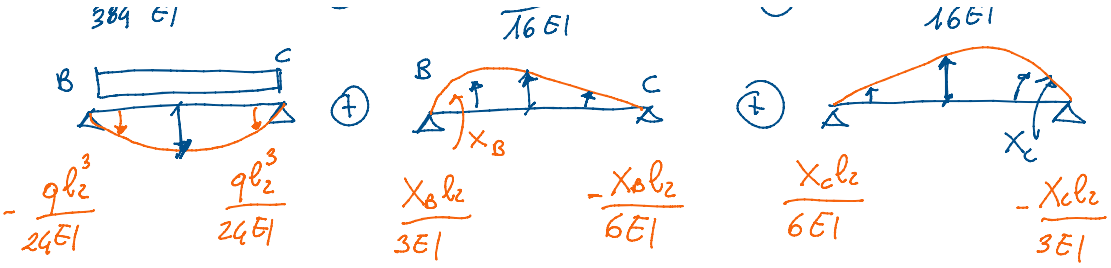
$2t = 2$

$s = 4$ $\Rightarrow$ $i = 2$



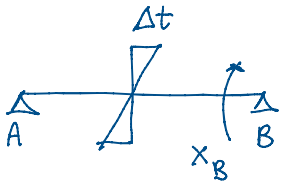
$$\begin{cases} \varphi_{BA} = \varphi_{BC} \\ \varphi_{CB} = \varphi_{CE} \end{cases}$$



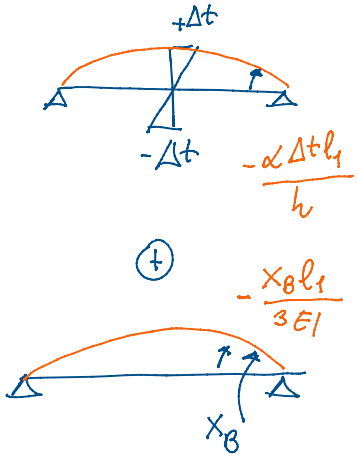


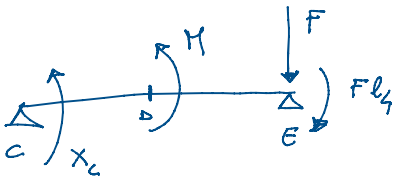
$$\varphi_{BC} = -\frac{ql_2^3}{24EI} + \frac{X_B l_2}{3EI} + \frac{X_C l_2}{6EI}$$

$$\varphi_{CB} = \frac{ql_2^3}{24EI} - \frac{X_B l_2}{6EI} - \frac{X_C l_2}{3EI}$$

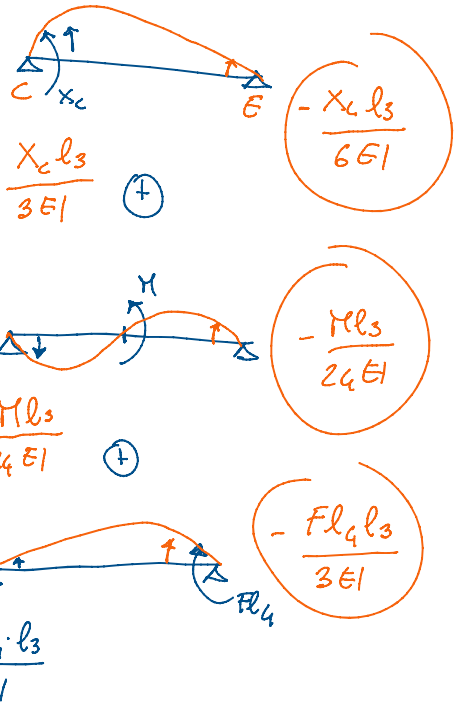


$$\varphi_{BA} = -\frac{\alpha \Delta t l_1}{h} - \frac{X_B l_1}{3EI}$$





$$\varphi_{CE} = \frac{X_C l_3}{3EI} - \frac{Ml_3}{24EI} + \frac{Fl_4 l_3}{6EI}$$



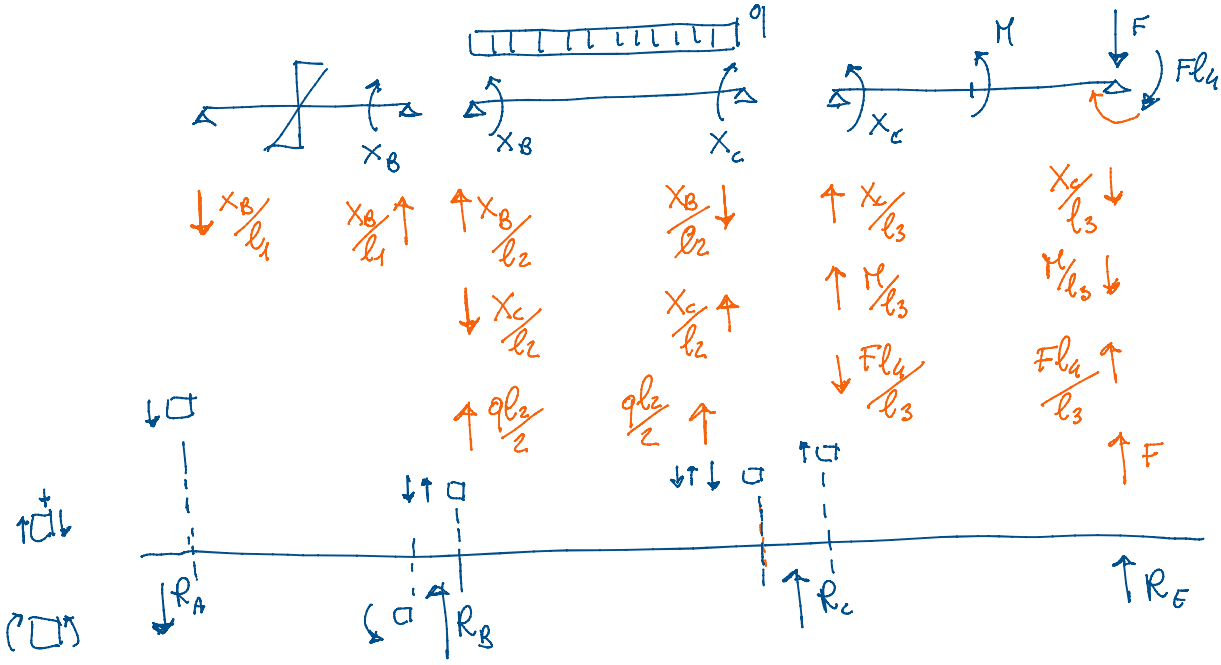
$$\begin{cases} -\frac{ql_2^3}{24EI} + \frac{X_B l_2}{3EI} + \frac{X_C l_2}{6EI} + \frac{\alpha \Delta t l_1}{h} + \frac{X_B l_1}{3EI} = 0 \\ \frac{X_C l_3}{3EI} - \frac{Ml_3}{24EI} + \frac{Fl_4 l_3}{6EI} - \frac{ql_2^3}{24EI} + \frac{X_B l_2}{6EI} + \frac{X_C l_2}{3EI} = 0 \end{cases}$$

$$\Rightarrow X_B = 31{,}32 \text{ kNm}$$

$$\Rightarrow X_C = 35{,}06 \text{ kNm}$$

$$\frac{}{3EI} - \frac{}{24EI} + \frac{}{6EI} - \frac{}{24EI} + \frac{}{6EI} - \frac{}{3EI}$$

② Reazioni vincolari



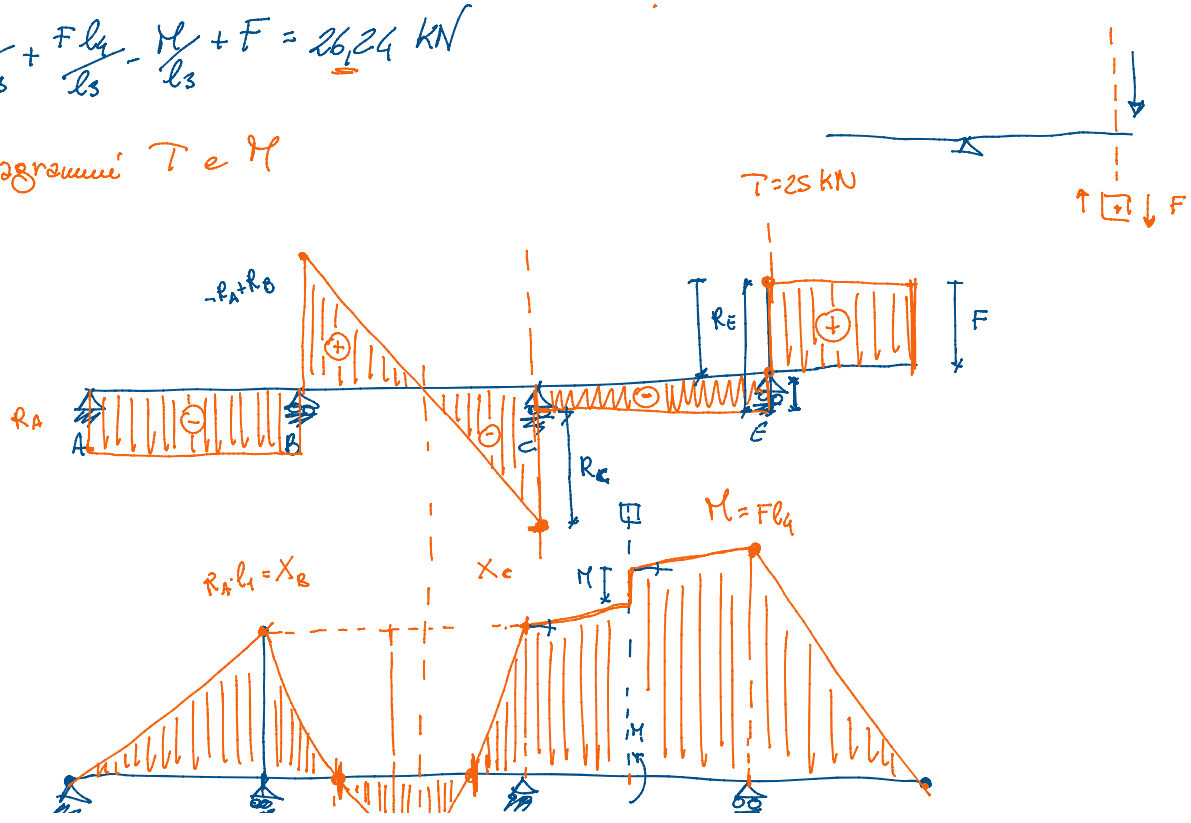
$$R_A = X_B/l_1 = 7{,}83 \text{ kN}$$

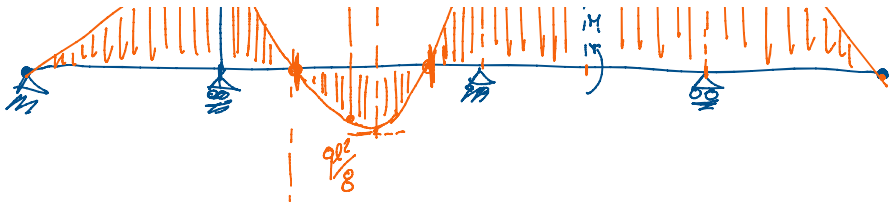
$$R_B = \frac{X_B}{l_1} + \frac{X_B}{l_2} - \frac{X_C}{l_2} + \frac{ql_2}{2} = 67{,}21 \text{ kN}$$

$$R_C = -\frac{X_B}{l_2} + \frac{X_C}{l_2} + \frac{ql_2}{2} + \frac{X_C}{l_3} - \frac{Fl_4}{l_3} + \frac{M}{l_3} = 59{,}39 \text{ kN}$$

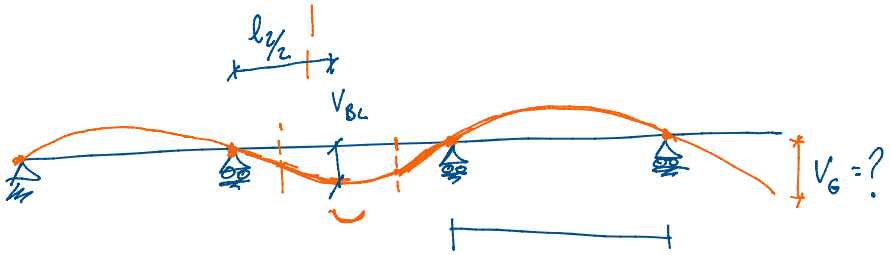
$$R_E = -\frac{X_C}{l_3} + \frac{Fl_4}{l_3} - \frac{M}{l_3} + F = 26{,}24 \text{ kN}$$

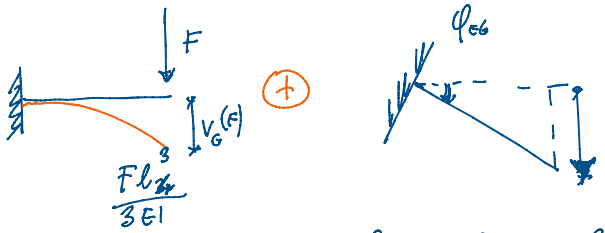
③ Diagrammi T e M





④ Andamento a mensola della linea elastica





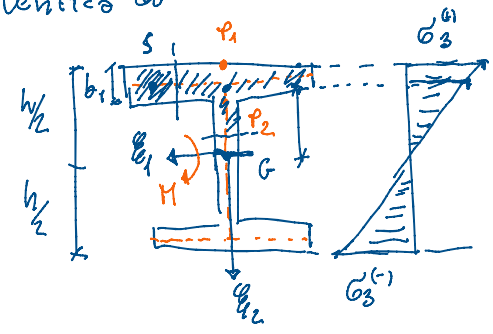
$$V_G = V_G^{(F)} + \varphi_{EG} \cdot l_4$$

$$\varphi_{EG} = \varphi_{EC} = -\frac{X_c l_3}{6EI} - \frac{M l_3}{24EI} - \frac{F l_4 l_3}{3EI} = -5{,}46 \cdot 10^{-3}$$

$$V_G = \frac{F l_4^3}{3EI} + 5{,}46 \cdot 10^{-3} \cdot l_4 = 0{,}0149 \text{ m} = 14{,}9 \text{ mm}$$

$$V_{BC} = \frac{5}{384}\frac{q l_2^4}{EI} - \frac{X_B l_2^3}{16EI} - \frac{X_c l_2^3}{16EI}$$

Verifica di resistenza

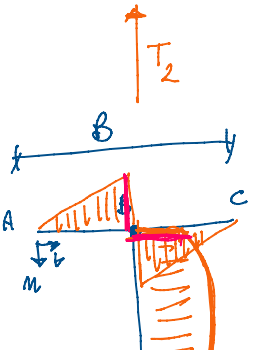


$T = -60{,}63$ kN

$M = 35{,}06$ kN

$$P_1 \Rightarrow \sigma_3 = \frac{M}{I} h/2 = \frac{M}{W} = 6514{,}53 \text{ N/cm}^2$$

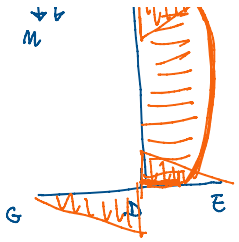
$$65{,}14 \text{ N/mm}^2$$



$$P_2 \Rightarrow \sigma_3 = \frac{M}{I}\left(\frac{h}{2} - b_2\right) = 65{,}14 \text{ N/mm}^2$$

tratto AB

$$\tau_{43} = -\frac{T_2 S_1'}{I \cdot T} = -\frac{T_2}{I \cdot T}\left(b_1 \cdot S\left(\frac{h}{2}\right)\right)$$



(P_1)

$$\tau_{tz} = -\frac{T_2 S_1'}{b_1 I_1} = -\frac{T_2}{b_1 I_1}\left(b_1 \cdot s\left(\frac{h}{2}\right)\right)$$

$s \downarrow B/2$

$$\tau_{tz}^{(P_1)} = -\frac{T_2}{I_1}\left(-\frac{h}{2}\right)\frac{B}{2} = -8{,}52\ N/mm^2$$

Tratto $B\Delta$

$$\tau_{tz} = -\frac{T_2 S_1'}{b_2 I_1} = -\frac{T_2}{b_2 I_1}\left[b_2 \cdot s\left[-\frac{H}{2}+\frac{s}{2}\right]+b_1 B\left(-\frac{H}{2}\right)\right] \Rightarrow \tau_{tz}^{(P_2)} = -30{,}71\ N/mm^2$$

$s \downarrow H/2$

Von Mises

$$\sigma_{eq}^{(P_1)} = \sqrt{\sigma_3^{(P_1)2} + 3\tau_{tz}^{(P_1)2}} = 67{,}38\ N/mm^2 < \sigma_{amm}$$

$\sigma_{amm} = 160$

Verificata

Tresca

$$\sigma_{eq}^{(P_1)} = \sqrt{\sigma_3^2 + 4\tau_{tz}^2} = 67{,}92\ N/mm^2\ (P_1)$$

Verificata