12/06/2019

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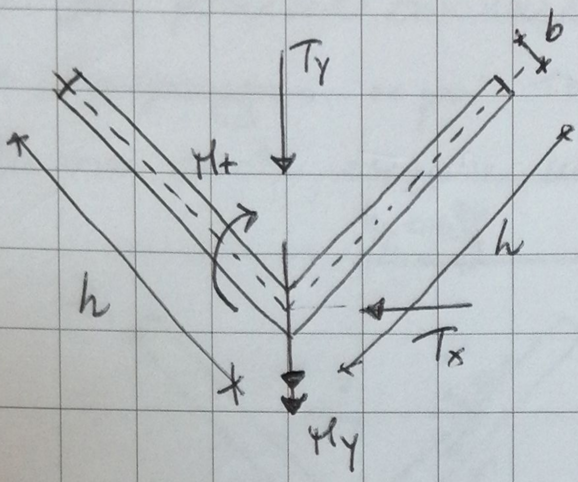
Esercizio n. 1 - Verifica di sicurezza



DATI:

$T_x = 20\ kN$

$T_y = 15\ kN$

$M_y = 2,5\ kNm$

$M_t = -0,05\ kNm$

$h = 100\ mm$

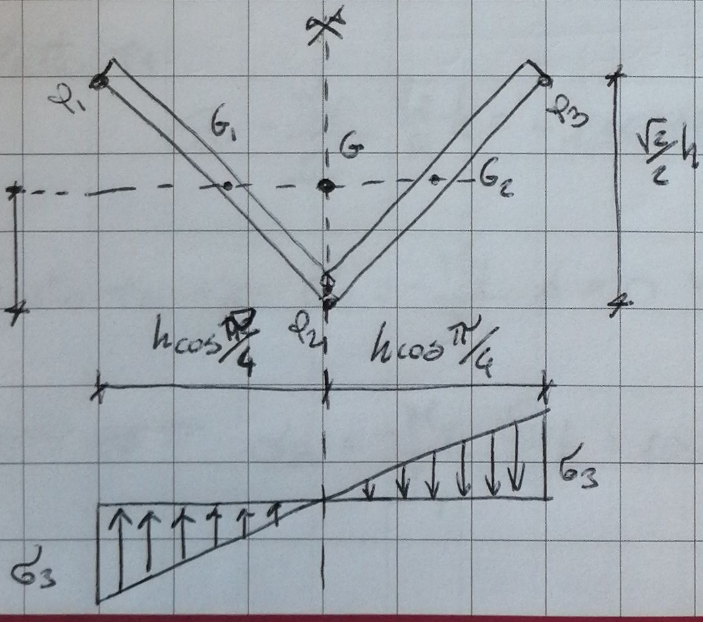
$b = 5\ mm$

$E = 210'000\ MPa$

$\sigma_{amm} = 160\ MPa$

Ricerca del baricentro

Il baricentro G si troverà sulla congiungente dei baricentri dei 2 rettangoli in cui si può scomporre la figura. ~~[illegible]~~ Si troverà anche sull'asse di simmetria della figura: quindi è univocamente determinato dalla loro intersezione



Tensioni normali σ_3

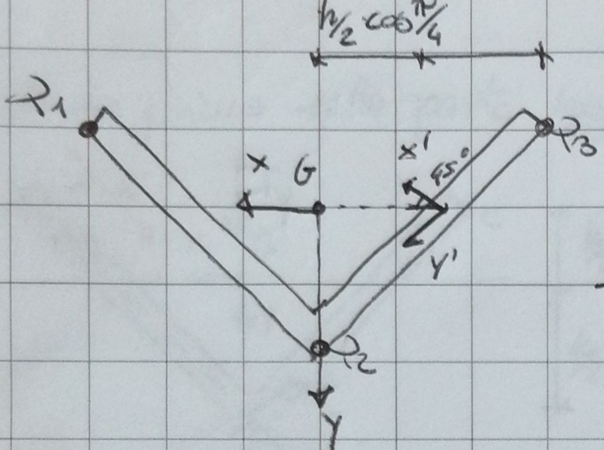
$$\sigma_3 = +\cancel{\frac{N}{A}} + \cancel{\frac{M_x}{I_x}y} - \frac{M_y}{I_y}x$$

$$\sigma_3 = -\frac{M_y}{I_y}x$$

É necessario calcolare I_y.

Si può procedere calcolando $I_{y'}$ dei singoli rettangoli che la compongono e poi trasportarli nel baricentro G.

Occorre anche eseguire una rotazione degli assi



$$I_{x'} = \frac{bh^3}{12} = \frac{5 \cdot 100^3}{12} = 416\,666{,}7 \ mm^4$$

$$I_{y'} = \frac{h \cdot b^3}{12} = \frac{100 \cdot 5^3}{12} = 1041{,}67 \ mm^4$$

Rotazione e trasporto dei 2 rettangoli

$$I_x = 2 \cdot \left[I_{x'} \cos^2 \frac{\pi}{4} + I_{y'} \sin^2 \frac{\pi}{4} + b \cdot h \cdot \phi^2 \right] = 417\,708{,}8 \ mm^4$$

ϕ → distanza tra x e x'

$$I_y = 2 \left[I_{y'} \cos^2 \frac{\pi}{4} + I_{x'} \cos^2 \frac{\pi}{4} + b \cdot h \cdot \left(\frac{\sqrt{2}}{2} \frac{h}{2} \right)^2 \right] = 1\,667\,708 \ mm^4$$

Punto P1

$$\sigma_3 = -\frac{M_y}{I_y} \cdot \frac{\sqrt{2}}{2} h = -106 \ MPa \quad (\text{compressione})$$

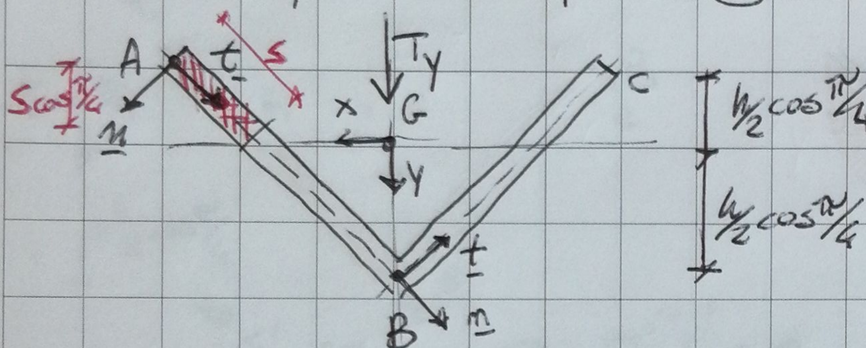
punto P2 ⇒ $\sigma_3 = -\frac{M_y}{I_y} \cdot \phi = 0 \ MPa$

punto P3 ⇒ $\sigma_3 = -\frac{M_y}{I_y} \cdot \frac{\sqrt{2}}{2} h = 106 \ MPa$ (trazione)

Per quanto riguarda il taglio si ha

$$\tau_{tz} = -\frac{T_x S'_y}{I_y b} - \frac{T_y S'_x}{I_x b}$$

Lavoriamo prima sulla parte legata a T_y $\Rightarrow$ $\tau_{tz}^{(T_y)} = \frac{-T_y S'_x}{I_x b}$



tratto AB $(0 \le s \le h)$

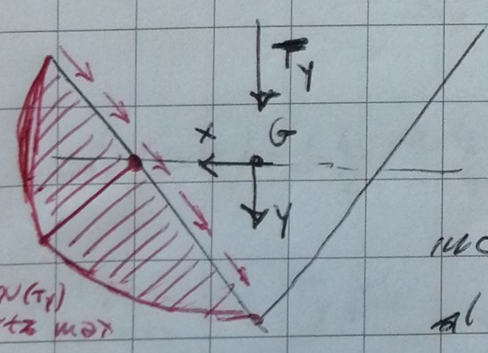
$$S'_x = -b \cdot s\left(\frac{h}{2} - \frac{s}{2}\right)\cos\frac{\pi}{4} \le 0 \quad \text{per} \quad s \le h$$

$$\Downarrow$$

$\tau_{tz}^{(T_y)} \ge 0$ (tensioni uscenti dal volume)

Calcolo $\tau_{tz,max}^{(T_y)}$ $\Rightarrow$ $\frac{dS'_x}{ds} = -\frac{bh}{2} + bs = 0$

$$\Downarrow$$

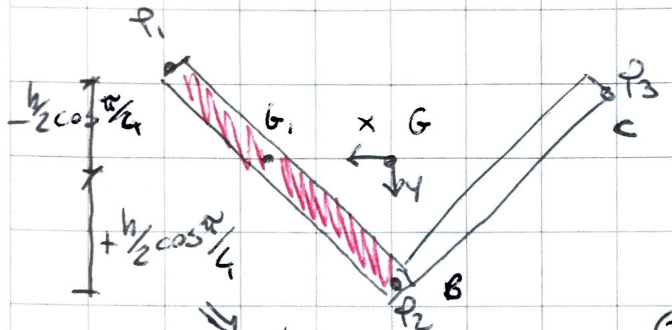


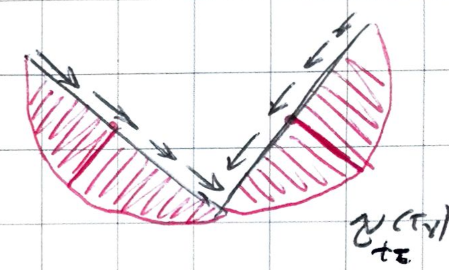
$s = \frac{h}{2}$ (il τ_{tz} max si attinge sulla fibra individuata dall'asse x baricentrico (ortogonale al taglio T_y) all'intersezione con la linea media della sezione)

$$S'_x\left(\frac{h}{2}\right) = -b \cdot \frac{h}{2}\left(\frac{h}{2} - \frac{h}{4}\right)\cos\frac{\pi}{4} = -4419{,}42 \text{ mm}^3$$

$$\tau_{tz,max}^{(T_y)} = -\frac{T_y\, S'_x}{I_x\, b} = +\frac{15000 \cdot 4419,42}{417\,708,8\, mm^4 \cdot 5m} = 31,74\ MPa$$

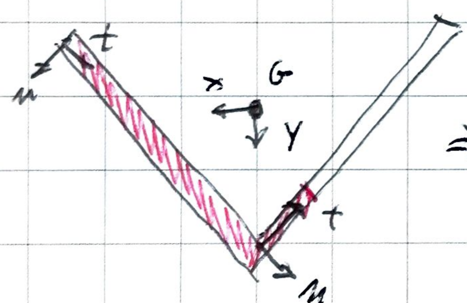
G_1 é sullo stesso asse di G quindi $S'_x = 0$



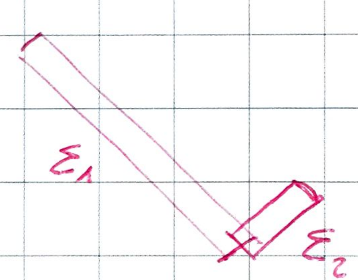


$$\Rightarrow S'_x = 0 \Rightarrow \tau_{tz}^{(T_y)} = \emptyset \text{ in } P_2$$

Tratto BC $(0 \le s \le h)$



$\Rightarrow$



$$S'_x = \underbrace{S'^{(\Sigma_1)}_x}_{0} + S'^{(\Sigma_2)}_x$$

$$S'_x = bs \cdot \left(\frac{h}{2} - \frac{s}{2}\right)\cos\frac{\pi}{4} \Rightarrow S'_x \ge 0 \text{ per } 0 \le s \le h$$

$$\Downarrow$$

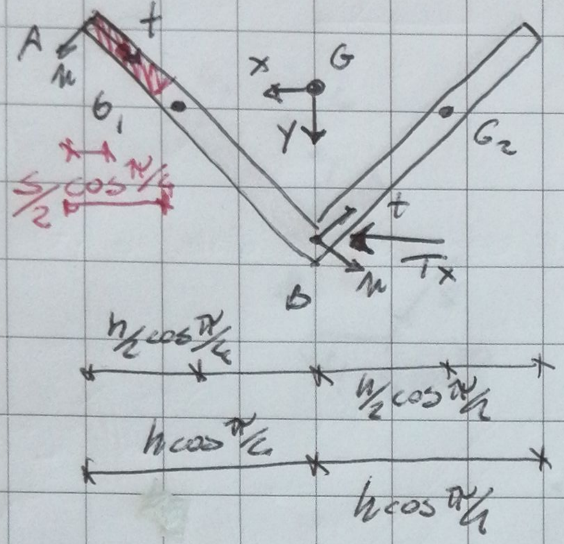
$$\tau_{tz}^{(T_y)} \le 0$$

(tensioni entranti nel volume)

$$S'_x\left(s = \frac{h}{2}\right) = b\frac{h}{2}\left(\frac{h}{2} - \frac{h}{4}\right)\cos\frac{\pi}{4} = 4419,42\ mm^3$$

$$\tau_{tz,max}^{(T_y)} = -\frac{T_y\, S'_x}{I_x\, b} = -31,74\ MPa$$

Passiamo al taglio T_x



$$\tau_{tz}^{(T_x)} = -\frac{T_x S'_y}{I_y b}$$

Tratto AB $(0 \le s \le h)$

$$S'_y = bs \cdot \left(h - \frac{s}{2}\right)\cos\frac{\pi}{4} \Rightarrow S'_y \ge 0 \text{ per } 0 \le s \le h \Rightarrow \tau_{tz} \le 0$$

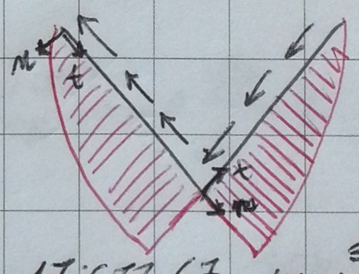
(tensioni entranti nel volume)

dove τ_{max}?

$$\frac{dS'_y}{ds} = bh\cos\frac{\pi}{4} - b\cos\frac{\pi}{4}\frac{2s}{2} = 0$$

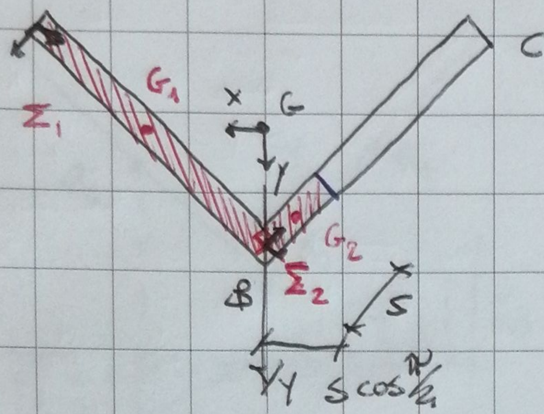
da cui $\Rightarrow s = h$

valore massimo



$$S'_y(s=h) = bh \cdot \left(h - \frac{h}{2}\right)\cos\frac{\pi}{4} = 17\,677{,}67 \ mm^3$$

$$\tau^{(T_x)}_{tz,max}(s=h) = -\frac{T_x S'_y}{I_y b} = -42{,}40 \text{ MPa}$$



Tratto BC $\quad 0 \le s \le h$

$$S'_y = S'^{(\Sigma_1)}_y + S'^{(\Sigma_2)}_y =$$

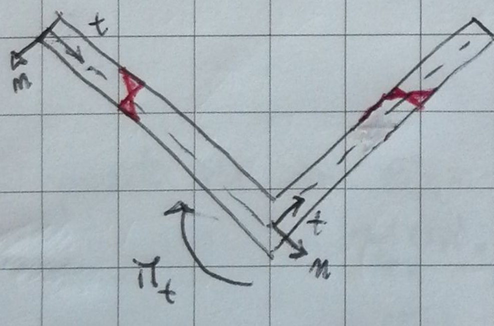
$$= \frac{bh^2}{2}\cos\frac{\pi}{4} - bs\cdot\frac{s}{2}\cos\frac{\pi}{4}$$

$$S'_y \ge 0 \Rightarrow \tau^{(T_x)}_{tz} \le 0$$

$$\frac{dS'_y}{ds} = bs\cos\frac{\pi}{4} = 0 \quad \text{per } s=0 \Rightarrow \tau^{(T_x)}_{tz}\ \max \text{ in } s=0$$

$$\tau^{(T_x)}_{tz} = -\frac{T_x S'_y}{I_y b} = -42,40\ \text{MPa}$$

Per quanto riguarda il momento torcente M_t

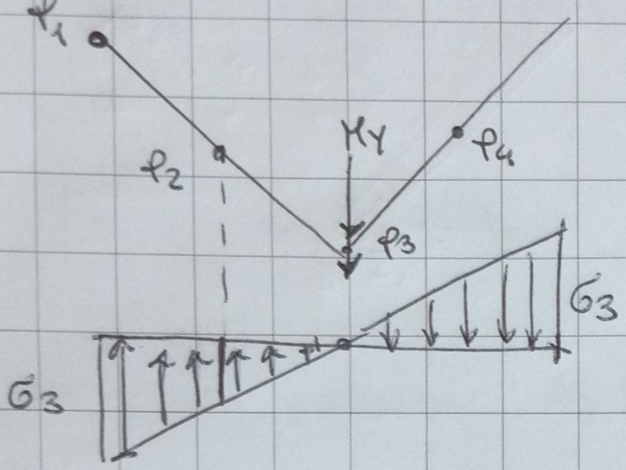


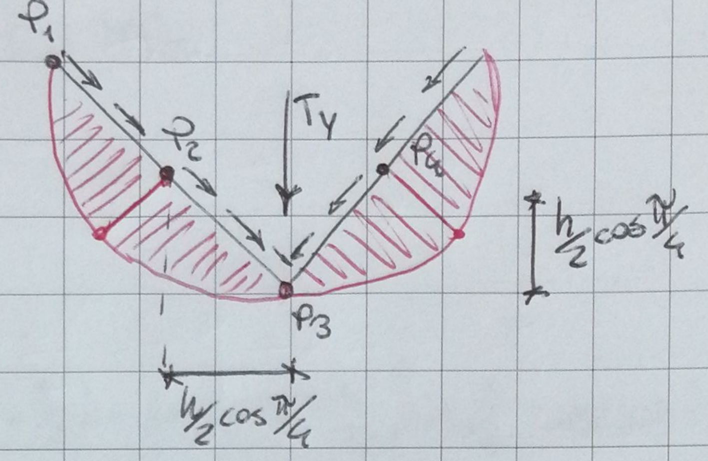
$$\tau^{(M_t)}_{tz} = -\cancel{2}\frac{M_t}{I_t}\cdot\frac{b}{\cancel{2}} = -\frac{M_t}{I_t}b$$

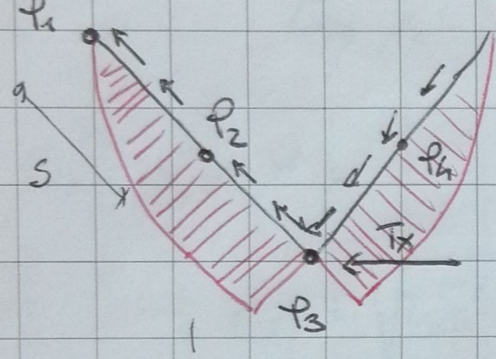
$$I_t = \sum_i I_{t,i} = 2\cdot\frac{1}{3}h\cdot b^3 = \frac{2}{3}hb^3$$

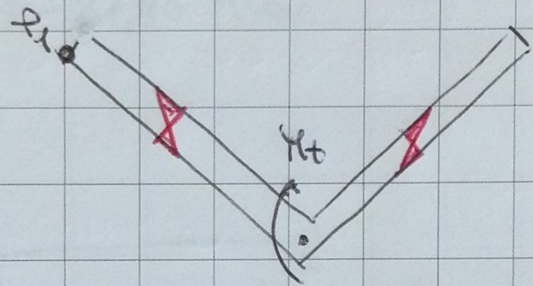
$$\tau^{(M_t)}_{tz} = -\frac{M_t}{\frac{2}{3}hb^{\cancel{3}2}}\cdot\cancel{b} = -\frac{-0,05\cdot10^6}{2/3\cdot100\cdot5^2} = 30\ \text{MPa}$$

In definitiva









Punti in cui eseguire la verifica: $\sigma_3(max) \Rightarrow P_1$

$\tau_{t3}(max) \Rightarrow P_2, P_3, P_4$

$\tau_{t3}(max) = \tau_{t3} = \tau_{t3}^{(T_x)} + \tau_{t3}^{(T_y)} + \tau_{t3}^{(M_t)}$

Nel punto $P_1 \Rightarrow \sigma_3 = -106$ MPa

$\tau_{tz}^{(T_y)} = \tau_{tz}^{(T_x)} = 0$

$\tau_{tz}^{(M_t)} = 30$ MPa

criterio di Tresca $\sigma_{eq} = \sqrt{\sigma_z^2 + 4\left(\tau_{t3}^{(T_x)} + \tau_{t3}^{(T_y)} + \tau_{t3}^{(M_t)}\right)^2}$

$\sigma_{eq} = \sqrt{-106^2 + 4 \cdot 30^2} = 121{,}80$ MPa < 160 MPa (σ_{adm})

Soddisfatta

In P_2:

$$\sigma_{3,P_2} = -\frac{M_Y}{I_Y} \cdot \frac{h}{2}\frac{\sqrt{2}}{2} = -53\ MPa$$

$$\tau^{(T_Y)}_{tz,P_2} = 31{,}74\ MPa$$

per $\tau^{(T_x)}_{tz,P_2}$ calcolare $S'_Y\left(s=\frac{h}{2}\right) = b \cdot \frac{h}{2}\left(h - \frac{h}{4}\right)\cos\frac{\pi}{4} = 13258{,}25\ mm^3$

$$\tau^{(T_x)}_{tz,P_2} = -\frac{T_x S'_Y}{I_Y b} = -31{,}80\ MPa$$

$$\tau^{(M_t)}_{tz} = 30\ MPa$$

$$\sigma_{eq} = \sqrt{-53^2 + 4\left(31{,}74 - 31{,}80 + 30\right)^2} = 79{,}97\ MPa < 160\ MPa$$

soddisfatta

$$\tau^{(P_2)}_{tz} = 31{,}74 - 31{,}80 + 30 = 29{,}94\ MPa$$

$$\tau^{(P_4)}_{tz} = -31{,}74 - 31{,}80 + 30 = -33{,}54\ MPa$$

Necessario verificare P_4

In P_4

$$\sigma_{eq} = \sqrt{53^2 + 4(-33{,}54)^2} = 85{,}49\ MPa < 160\ MPa \quad \text{ok!}$$

In P_3

$\sigma_z = \emptyset \qquad \tau_{tz} = \emptyset - \tau^{(T_x)}_{tz\,max} + 30\ MPa = -42{,}40\ MPa + 30\ MPa =$

$= -12{,}40\ MPa$

$$\sigma_{eq} = \sqrt{4(-12{,}40)^2} = 24{,}80\ MPa < 160\ MPa\ (OK)$$