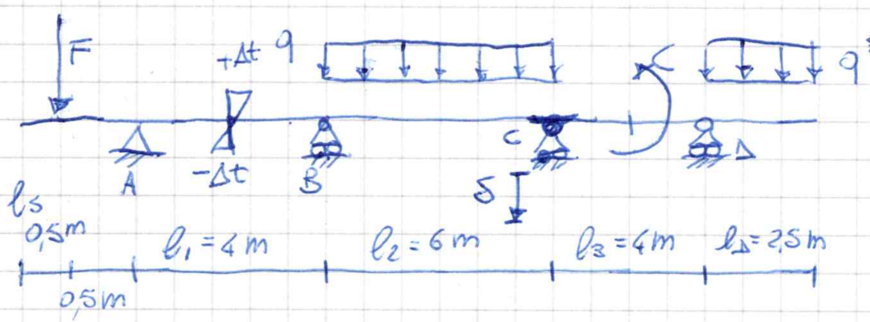
1

Trave continua



$F = 120\ KN$

$C = 10\ KNm$

$q = 20\ KN/m$

$q^* = 16\ KN/m$

$\Delta t = 8{,}46\ °C$

$h = 200\ mm$

$I = 3600\ cm^4$

$E = 2{,}1 \cdot 10^5\ N/mm^2$

$\alpha = 1{,}2 \cdot 10^{-5}\ °C^{-1}$

$\delta = 5 \cdot 10^{-4}\ m$

Non vi sono forze orizzontali, quindi il problema estensionale è disaccoppiabile dal problema flessionale, e in questo caso da propria soluzione banale. = 0

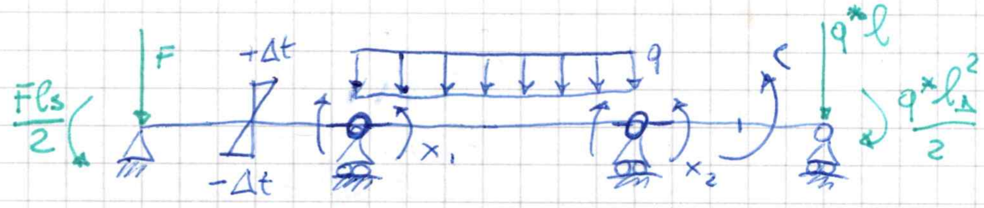
$2t = 2$

$s = 4 \Rightarrow i = 2$

Con il problema elastostatico risolvere la struttura è troppo lungo, quindi si usa il metodo delle forze.

Non è utile operare sui vincoli esterni, quindi si opera declassando gli incastri interni in modo da avere più tronchi riconducibili a schemi già noti.

È anche possibile eliminare gli sbalzi.



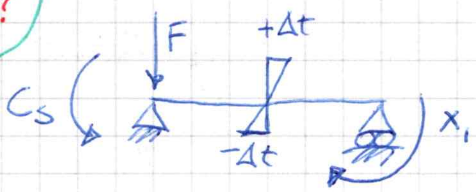
in inseriscono cerniere interne

schema isostatico principale.

eq. di congruenza

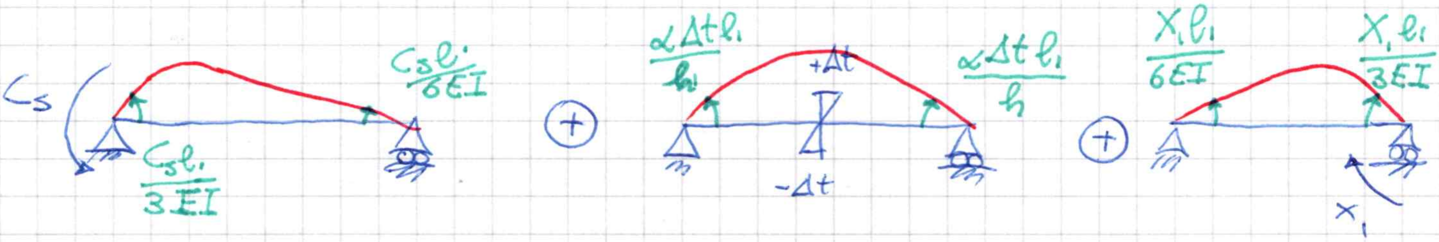
$$\begin{cases} \Delta\varphi_B = \varphi_{BA} - \varphi_{BC} = 0 \\ \Delta\varphi_C = \varphi_{CD} - \varphi_{CB} = 0 \end{cases}$$

$\varphi_{BA} = ?$



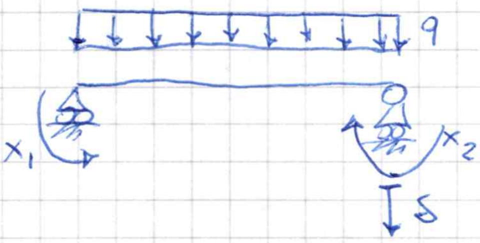
la F non da rotazioni perché è assorbita dal vincolo

$$\varphi_{BA} = \varphi_{BA}^{(C_s)} + \varphi_{BA}^{(\Delta t)} + \varphi_{BA}^{(X_1)}$$

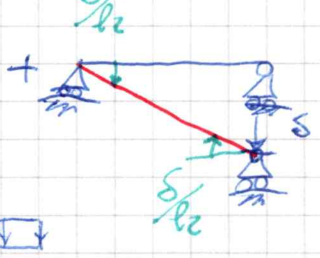


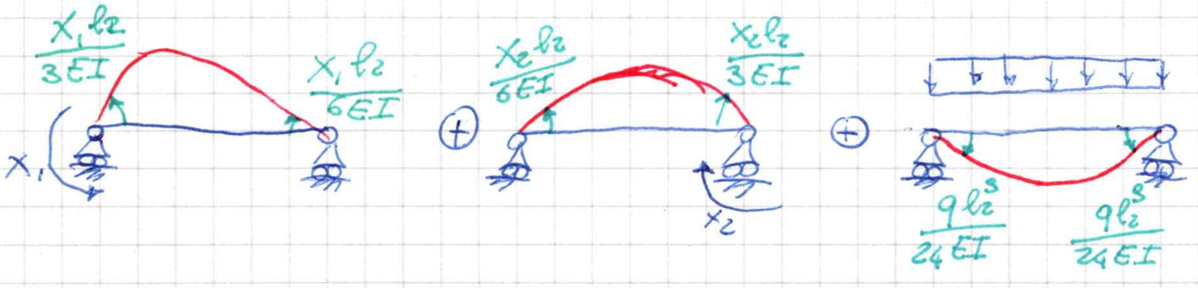
$$\varphi_{BA} = -\frac{C_s l_1}{6EI} - \frac{\alpha \Delta t\, l_1}{h} - \frac{X_1 l_1}{3EI}$$

φ_{BC}



$$\varphi_{BC} = \varphi_{BC}^{(X_1)} + \varphi_{BC}^{(X_2)} + \varphi_{BC}^{(q)} + \varphi_{BC}^{(\delta)}$$





$$\varphi_{BC} = \frac{X_1 l_2}{3EI} + \frac{X_2 l_2}{6EI} - \frac{q l^3}{24EI} - \frac{\delta}{l_2}$$

Quindi la prima eq. diventa.

$$\begin{cases} \varphi_{BA} - \varphi_{BC} = \frac{X_1 l_2}{3EI} + \frac{X_2 l_2}{6EI} - \frac{q l^3}{24EI} - \frac{\delta}{l_2} + \frac{C_S l_1}{6EI} + \frac{\alpha \Delta t l_1}{h} + \frac{X_1 l_1}{3EI} = 0 \end{cases}$$

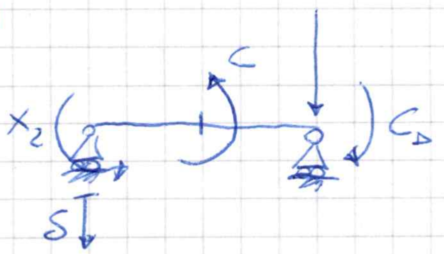
φ_{CB}

$$\varphi_{CB} = \varphi_{CB}^{(X_1)} + \varphi_{CB}^{(X_2)} + \varphi_{CB}^{(q)} + \varphi_{CB}^{(S)}$$

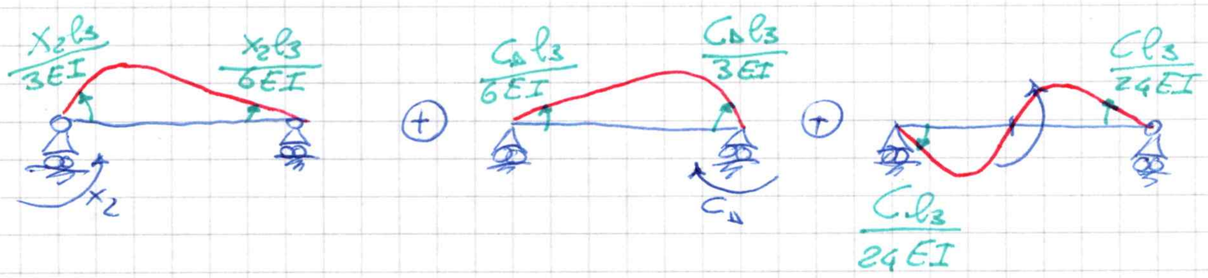
La trave è già disegnata per l'angolo φ_{BC}

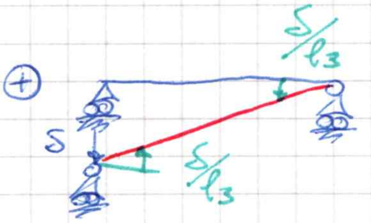
$$\varphi_{CB} = -\frac{X_1 l_2}{6EI} - \frac{X_2 l_2}{3EI} + \frac{q l^3}{24EI} - \frac{\delta}{l_2}$$

φ_{CD}



$$\varphi_{CD} = \varphi_{CD}^{(X_2)} + \varphi_{CD}^{(C_D)} + \varphi_{CD}^{(S)} + \varphi_{CD}^{(C)}$$





$$\varphi_{CD} = \frac{X_2 l_3}{3EI} + \frac{C_D l_3}{6EI} - \frac{C l_3}{24EI} + \frac{S}{l_3}$$

la seconda equazione diventa

$$\varphi_{CD} - \varphi_{CB} = \frac{X_2 l_3}{3EI} + \frac{C_D l_3}{6EI} - \frac{C l_3}{24EI} + \frac{S}{l_3} + \frac{X_1 l_2}{6EI} + \frac{X_2 l_2}{3EI} - \frac{q l^3}{24EI} + \frac{S}{l_2} = 0$$

Le incognite iperstatiche sono X_1 e X_2

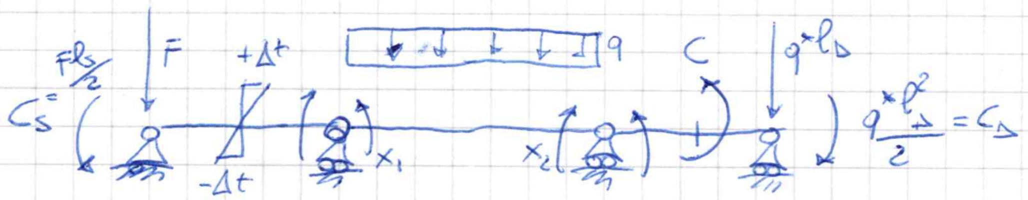
Risolvendo i valori numerici si ha il sistema lineare

$$\begin{cases} 4{,}41 \cdot 10^{-7} X_1 + 1{,}32 \cdot 10^{-7} X_2 = 1{,}68 \cdot 10^{-2} \\ 1{,}32 \cdot 10^{-7} X_1 + 4{,}41 \cdot 10^{-7} X_2 = 1{,}88 \cdot 10^{-2} \end{cases}$$

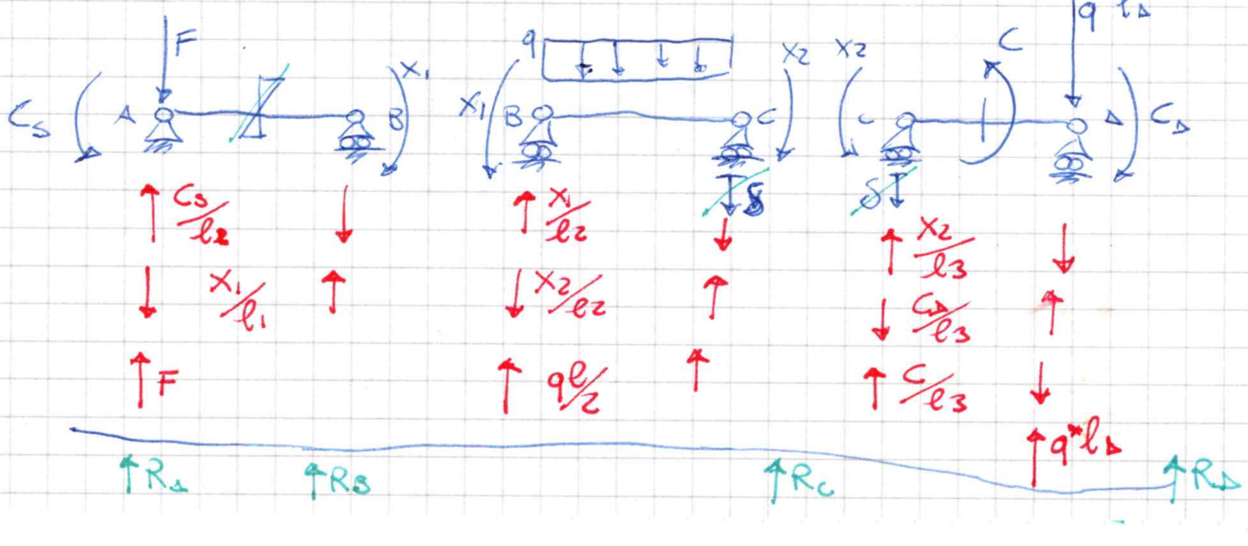
se i calcoli sono giusti la matrice dei termini noti è simmetrica.

$X_1 = 27{,}90\ KNm$

$X_2 = 34{,}21\ KNm$



Determinare le reazioni vincolari



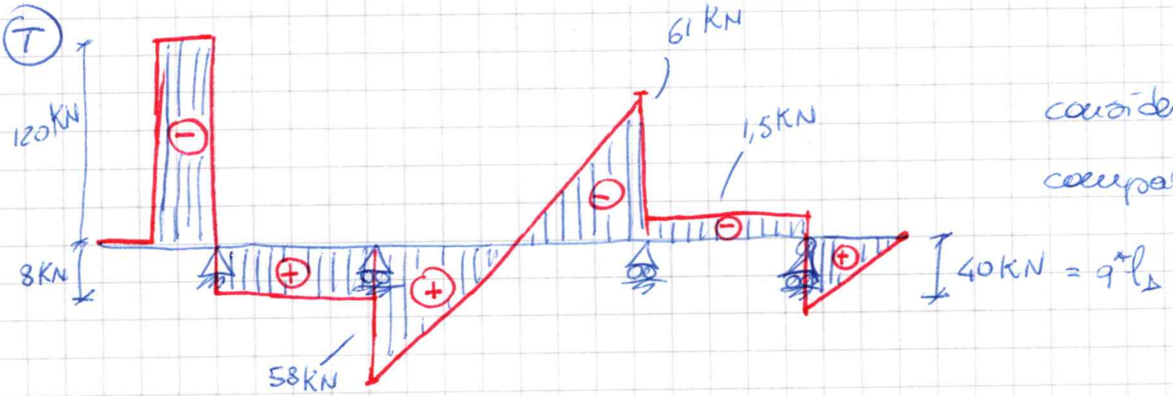
$$R_A = \frac{C_S}{l_1} - \frac{X_1}{l_1} + F$$

$$R_B = -\frac{C_S}{l_1} + \frac{X_1}{l_1} + \frac{X_1}{l_2} - \frac{X_2}{l_2} + \frac{ql}{2}$$

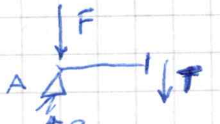
$$R_C = -\frac{X_1}{l_2} + \frac{X_2}{l_2} + \frac{ql}{2} + \frac{X_2}{l_3} - \frac{C_D}{l_3} + \frac{C}{l_3}$$

$$R_D = -\frac{X_2}{l_3} + \frac{C_D}{l_3} - \frac{C}{l_3} + q^* l_D$$

Diagrammi (T. M. V



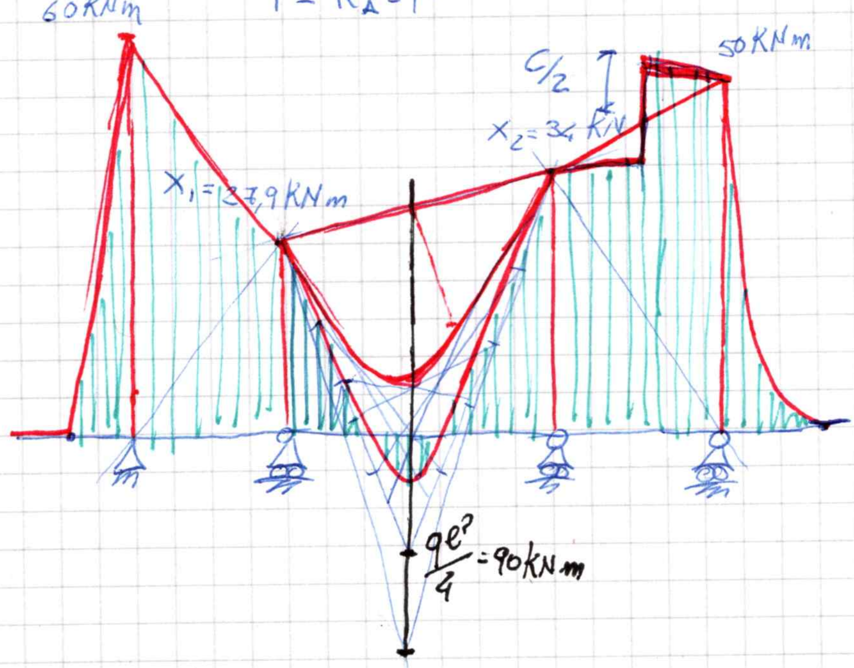
considerando le campate separate



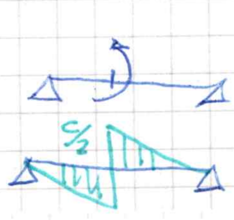
$$F + T - R_A = 0$$

$$\Downarrow$$

$$T = R_A - F$$



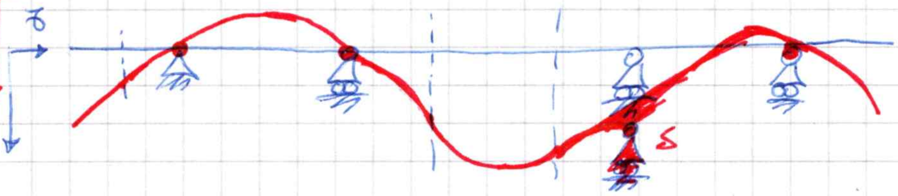
si considerano ora il carico q e il momento C riferendoci alla fondamentale del momento.



V

punti di flesso presi dal momento

$$\boxed{\frac{d^2v}{dz^2} = -\frac{M}{EI}}$$



$$\theta + \theta^* = -\frac{d^2v}{dz^2}$$

$$\boxed{-\frac{d^2v}{dz^2} = \frac{M}{EI} + \theta^*}$$

$$\theta^* = \frac{2\alpha\Delta t}{h} \qquad \begin{matrix} -\Delta t \\ +\Delta t \end{matrix}$$

$$\left.\frac{d^2v}{dz^2}\right|_{z=z_A} = -\frac{M(z_A)}{EI} - \theta^* = \frac{60\,kN}{EI} + \frac{2\alpha\Delta t}{h} > 0$$

$$\left.\frac{d^2v}{dz^2}\right|_{z=z_B} = \frac{27}{EI} + \frac{2\alpha\Delta t}{h} > 0$$

la curvatura è ancora positiva.